On the control of maxplus linear system subject to state restriction

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Abstract

This paper deals with the control of discrete events systems subjected to synchronization and time delay phenomena, which can be described by using the max-plus algebra. The objective is to design a feedback controller that guarantees that the system evolves without violating time restrictions imposed to the state. To this end an equation is derived, which involves the system, the feedback and the restriction matrices. In addition conditions concerning the existence of the feedback are discussed and sufficient conditions that ensure the computation of the feedback is presented. To illustrate the contribution of this paper, a workshop control problem is presented, for which a controller is designed in order to guarantee that time restrictions are respected.

Key words:
Discrete Event Systems, Timed-event graphs, Max-plus Algebra.
PACS:

1 Introduction

Many engineering systems such as manufacturing, transport and communication networks and others, can be modeled by using the Discrete Event Systems (DES) framework (Cassandras and Lafortune, 1999). The Max-plus algebra is particularly suitable to represent DES that are subjected to synchronization and delay phenomena, because the inherent non-linearities of such systems are written as linear equations in this algebraic context (Baccelli et al., 1992). This kind of system can be represented graphically by using Timed Event Graph (TEG),

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which is a particular class of timed Petri net in which all places have single upstream and single downstream transitions (Murata, 1989). From this point many results have been achieved concerning, not only the analysis problem (performance evaluation), but also the control problem. Concerning the control, many different problems have been treated in the literature. For instance, in Menguy et al. (2000) it is proposed a control strategy when some inputs are unknown; Cottenceau et al. (2001) have proposed a closed loop model reference control. In Lüders and Santos-Mendes (2002) it is proposed a multivariable control and Lhommeau et al. (2004) have considered parameter uncertainties using interval analysis. The model reference control based on precompensation and feedback is presented in Maia et al. (2003) and Maia et al. (2005). Finite horizon control problems for uncertain system was addressed in Necoara et al. (2007).

This paper deals with a control problem for max-plus linear problem, for which the objective is to find a control law that ensures that system state evolves without violation of some pre-defined restrictions. This problem is based on the ideas of Ouerghi et al. (2005); Amari et al. (2005); Ouerghi and Hardouin (2006); Garcia (2007); Katz (2007); Houssin et al. (2007). The proposed approach is based uniquely on the algebraic property of the matrices and the controller matrix is obtained by solving a maxplus linear equation (Cuninghame-Green and Butkovic, 2003). Moreover conditions concerning the existence of the feedback are discussed and sufficient conditions to compute the feedback are presented. The problem of the closed-loop system stability is also addressed.

The paper organization is as follows. Section 2 introduces some algebraic tools concerning the idempotent semiring and Residuation theories and their applications to max-plus linear systems. Section 3 presents the control problem and some theoretical results. Numerical results for a workshop problem are shown in section 4. A conclusion is given in section 5.

2 Mathematical Tools

The dynamic behavior DES subjected to synchronization and delay phenomena can be described by using the maxplus algebra, defined by using a set $\mathbb{D}$ and the operations $\oplus$ and $\otimes$. The operation $\oplus$ is associative, commutative and idempotent, that is, $a \oplus a = a, \forall a \in \mathbb{D}$. The operation $\otimes$ is associative and distributive at left and at right with respect to $\oplus$. For instance, if one considers the set $\mathbb{Z} \cup \{-\infty\}$, both operations have neutral elements given by: $\varepsilon = -\infty$ for $\oplus$ and $e = 0$ for $\otimes$ (ε and e are the usual notation for them). Moreover, $\forall a$, $a \otimes \varepsilon = \varepsilon \otimes a = \varepsilon$, that is, ε is absorbing with respect to $\otimes$. In general, any set $\mathbb{D}$ equiped with two internal operations ($\oplus$ and $\otimes$) satisfying all these properties is called an idempotent semiring or dioid, denoted by $(\mathbb{D}, \oplus, \otimes)$. Clearly $(\mathbb{Z} \cup \{-\infty\}, max, +)$ is an idempotent semiring, hereafter called $\overline{\mathbb{Z}}_{\max}$. In an idempotent semiring, a partial order relation is defined by $b \preceq a \text{ iff } a = a \oplus b$ and $x \wedge y$ denotes the greatest lower bound between x and y. An idempotent semiring $\mathbb{D}$ is said to be complete if it is closed for infinite $\oplus$-sums and if $\otimes$ distributes over infinite $\oplus$-sums. Most of the time the symbol $\otimes$ will be omitted as in conventional algebra, moreover $a^i = a \otimes a^{i-1}$ and $a^0 = e$.

In general, the state evolution of a maxplus linear system can be described by the following equations:

$$x(k) = Ax(k) \oplus Bu(k) \tag{1}$$

in which vectors $x(k) \in (\overline{\mathbb{Z}}_{\max})^n$, $u(k) \in (\overline{\mathbb{Z}}_{\max})^p$ and $y(k) \in (\overline{\mathbb{Z}}_{\max})^m$ represent respectively the date of k^{th} firing of the state, input and output transitions. A, B, C are the system matrices of appropriate dimensions. One recalls that, since $x(k) \preceq x(k+1)$, that is, the firing dates are nondecreasing, then $I \preceq A$, in which I is an identity matrix. It is important to remark that maxplus linear systems can be handled by using Scilab toolboxes, which can be downloaded from the sites (J-P.Quadrat, 2007; Hardouin et al., 2007).

Dealing with the inequality $EX \oplus G \preceq X$ is a relevant issue in many max-plus linear problems and an important result is presented in following theorem which is adapted from Baccelli et al. (1992).

Theorem 1 *The inequality $EX \oplus G \preceq X$ defined over a complete idempotent semiring $\mathbb{D}$ is equivalent to $X = E^*X \oplus E^*G$ where $E^* = \bigoplus_{i \in \mathbb{N}} E^i$ (Kleene star operator). As a consequence, one can see that $X = E^*X$ and E^*G is the least element that satisfies the presented inequality.*

Remark 1 *One remarks that if $X \in (\overline{\mathbb{Z}}_{\max})^n$, and E has negative circuit weights, then $X = EX \oplus G \Leftrightarrow X = E^*G$. In this paper, it is assumed that this condition for the matrix E is always satisfied* [1].

Control synthesis deals with the inversion of mappings by solving equations. Mappings defined over an idempotent semiring, in general, do not admit inverse, however the residuation theory allows to characterize the solution set of an inequality such that $f(x) \preceq y$, which is useful in many control problems. The reader may consult Blyth and Janowitz (1972) to obtain a complete presentation of this theory.

Definition 1 (Residuated mapping) *An isotone mapping $f : \mathcal{D} \to \mathcal{E}$, where $\mathcal{D}$ and $\mathcal{E}$ are partially ordered sets, is a residuated mapping if for all $y \in \mathcal{E}$ there exists a greatest element x that satisfies the inequality $f(x) \preceq y$ (hereafter denoted $f^\sharp(y)$). The mapping $f^\sharp$ is called the residual of f.*

The mappings $L_a : x \mapsto a \otimes x$ and $R_a : x \mapsto x \otimes a$ defined over a complete idempotent semiring $\mathbb{D}$ are both residuated Baccelli et al. (1992). Their residuals are isotone mappings [2] denoted respectively by $L_a^\sharp(x) = a \backslash\!\!\!\circ\, x$ and $R_a^\sharp(x) = x /\!\!\!\circ\, a$.

Dually , if there exists a least element x for the inequality $y \preceq f(x)$ it is denoted by $f^\flat(y)$ and named as dual residual of $f^\flat(y)$. The mapping f is called the *dual residual* of f. For instance, the function $T(x) = x \oplus a$, defined over a complete idempotent semiring $\mathbb{D}$, is dually residuated, and its residual is denoted by $T^\flat(x) = x \ominus a$.

In the following we present an important result to solve result equations defined in the idempotent semiring $\overline{\mathbb{Z}}_{\max}$.

2.1 Solving equations in $\overline{\mathbb{Z}}_{\max}$

To solve equations, it is important to remark that a set of linear equation of the type $\{z | A \otimes z \preceq b\}$ in complete idempotent semiring always have a greatest element, which is given by the residuation theory. This el-

[1] One remark that if E has positive circuit weights then it has elements equal to infinity and this situation has no practical interest.

[2] f is isotone mapping if it preserves order, that is, $a \preceq b \Longrightarrow f(a) \preceq f(b)$.

ement is given by:

$$z = A \backslash b, \tag{2}$$

In the idempotent semiring $\overline{\mathbb{Z}}_{\max}$, $A \backslash b = A^{\Delta} \otimes' b$ in which $A^{\Delta} = [-a_{ji}]$ and $\otimes'$ is defined as $\wedge$ operator (min operator).

If a finite solution for the linear system $A \otimes z = B \otimes y$ exists, one can use the following algorithm, which can provide a solution in finite number of step. For more detail concerning convergence issues see Cuninghame-Green and Butkovic (2003).

Begin
Choose arbitrary finite element z
Set $r = 0$***;*** $z(0) = z$
Repeat
$\mapsto y(r) = B \backslash (A \otimes z(r))$***;***
$\mapsto z(r) = A \backslash (B \otimes y(r))$***;***
$\mapsto r = r + 1$;
Until a tolerance is achieved
End

In Cuninghame-Green and Butkovic (2003), by considering that A and B has at least one finite element on each row and on each column, it is shown that the presented algorithm converges if and only if a finite solution to $A \otimes z = B \otimes y$ exists.

3 Proposed Control approach

In this section the control problem is presented and a sufficient condition for the controller existence is given.

Definition 2 (Control problem) *The aim is to find a feedback controller for the maxplus linear system:*

$$x(k) = Ax(k-1) \oplus Bu(k), \tag{3}$$

in order to ensure that the state evolution respects the following constraint:

$$Ex(k) \preceq x(k), \tag{4}$$

where $E \in \overline{\mathbb{Z}}^{n \times n}_{max}$ *is a restriction matrix with negative circuit weights.*

Obviously, depending on the characteristics of the matrices A, B and E the control problem can not be solve. This is illustrated by the example 1.

Example 1 *It is easy to see that the following system:*

$$x_1(k) = x_1(k-1), \tag{5}$$

$$x_2(k) = 10x_1(k-1) \oplus x_2(k-1) \oplus u(k), \tag{6}$$

can not ensure the restriction $x_2(k) \preceq x_1(k)$*.*

First of all, to solve the control problem, one can use the theorem 1 to obtain the following equivalence for the restriction 4:

$$Ex(k) \preceq x(k) \Leftrightarrow E^*x(k) = x(k), \quad \forall k \geq 0. \tag{7}$$

This equation means that the state evolves such as $x(k) \in \mathsf{Im}E^*$. Concerning the initial condition, one have the following property.

Property 1 *The initial condition for the control problem is feasible if and only if* $x(0) \in \mathsf{Im}E^*$*.*
Proof:

If the initial condition is feasible, then $x(0) = E^*x(0)$*, therefore* $\exists v$ *such that* $x(0) = E^*v$*.*

If $x(0) = E^*v$ then $E^*x(0) = E^*E^*v = E^*v = x(0)$, therefore the initial condition is feasible. ∎

To solve the control problem, the proposed approach consists in finding a feedback control law such that $u(k) = Fx(k-1)$, in which $F \in \overline{\mathbb{Z}}_{max}^{p\times n}$, in order to ensure that $E^*x(k) = x(k)(\forall k \geq 0)$. From Eq. 3 the system state can be written :

$$x(k) = (A \oplus BF)x(k-1), \quad \forall k \geq 1. \tag{8}$$

Proposition 1 *For all feasible initial conditions, i.e. $\forall x(0) \in \mathsf{Im}E^*$, F is a solution for the control problem if and only if*

$$E^*(A \oplus BF)E^* = (A \oplus BF)E^* \tag{9}$$

Proof:

According to Eq. 8 the controller F must be such that $E^*x(k) = x(k)$, then by rewriting equation 7, F must be such that :

$$E^*(A \oplus BF)x(k-1) = (A \oplus BF)x(k-1), \quad \forall k \geq 1 \tag{10}$$

In particular, for $k = 1$:

$$E^*(A \oplus BF)x(0) = (A \oplus BF)x(0), \tag{11}$$

In which $x(0)$ is a feasible initial condition. According to property 1 this equation can be written as follows:

$$E^*(A \oplus BF)E^*v = (A \oplus BF)E^*v. \tag{12}$$

As this equation must be hold for all feasible condition, that is $\forall v \in \overline{\mathbb{Z}}_{max}^{n}$, one must guarantee that :

$$E^*(A \oplus BF)E^* = (A \oplus BF)E^*. \tag{13}$$

This development has shown the necessity. The sufficiency is presented in the following.

If Eq.9 is true then $E^*x(1) = x(1)$, since $\exists v$ s.t. $x(0) = E^*v$, and:

$$\begin{aligned} E^*(A \oplus BF)E^* = (A \oplus BF)E^* \Rightarrow \\ E^*(A \oplus BF)E^*v = (A \oplus BF)E^*v \Rightarrow \\ E^*x(1) = x(1). \end{aligned} \tag{14}$$

Furthermore, by assuming that $E^*x(k) = x(k)$ and if Eq. 9 is true :

$$\begin{aligned} E^*(A \oplus BF)E^* = (A \oplus BF)E^* \Rightarrow \\ E^*(A \oplus BF)E^*x(k) = (A \oplus BF)E^*x(k) \Rightarrow \\ E^*x(k+1) = x(k+1). \end{aligned} \tag{15}$$

Reasoning by induction, if Eq. 9 is true then the controller F ensures that the state evolution $x(k+1) = (A \oplus BF)x(k)$ is such that $E^*x(k) = x(k)$ $\forall k \geq 0$.

∎

Sometimes, it is more interesting to work with inequalities instead of equalities. In this sense, proposition 2 gives an equivalence for the Eq.9.

Proposition 2 *Finding a solution F for the Eq.9 is equivalent to find F that satisfies the following inequalities :*

$$(E^*A \ominus AE^*) \preceq BFE^* \tag{16}$$

$$F \preceq (E^*B) \diagdown (AE^* \oplus BFE^*) \quad (17)$$

Proof:

By the dual residuation, 16 is equivalent to $E^*A \preceq AE^* \oplus BFE^*$. The residuation ensures that 17 is equivalent to $E^*BF \preceq AE^* \oplus BFE^*$. Therefore 16 and 17 together are equivalent to:

$$E^*(A \oplus BF) \preceq (A \oplus BF)E^*. \quad (18)$$

Finally, one can see that this inequality is equivalent to $E^*(A \oplus BF)E^* \preceq (A \oplus BF)E^*$. As $(A \oplus BF)E^* \preceq E^*(A \oplus BF)E^*$ is always true, then the inequality 18 results in $E^*(A \oplus BF)E^* = (A \oplus BF)E^*$. On the other hand, this equation implies in the inequality 18, which leads to an equivalence.

■

Remark 2 *The result given by the proposition 2 can be useful to find a solution for the equation Eq. 9. One can see that if $A \in \mathsf{Im}E^*$ then $F \preceq B \diagdown A$ is a solution for the problem, since for this condition $E^*A \preceq AE^*$ and so $E^*A \ominus AE^*$ is a null matrix, that is, this restriction is always respected. Moreover, since residuation ensures that $BF \preceq A$, one can see that inequality 17 is also respected.*

On the other hand one can see from the inequality 16 that if B has a null row there exists a feedback with non null entries for the proposed control problem only if the correspondent row in $(E^*A \ominus AE^*)$ is also null. One can see, for instance, that this is not the case of the example 1. This fact has motivated the use of the following definition, which was adapted from Cuninghame-Green and Butkovic (2003).

Definition 3 (G-astic Matrix) *A matrix is G-astic if it has at least one finite element in each row.*

Lemma 1 *If B is G-astic, it is always possible to choose a matrix Z, by making its elements large enough, such that $L \preceq BZ$, $\forall L \in \overline{\mathbb{Z}}_{max}^{n\times n}$.*

Proof: It comes directly from the following fact:

If B is G-astic then $((\forall i)(\exists l)|B_{il} \neq \varepsilon)$. As $(BZ)_{ij} = \bigoplus_{l=1}^{p} B_{il} \otimes Z_{lj}$, then by choosing the Z elements large enough, it is always possible to make $L_{ij} \preceq (BZ)_{ij}$. ■

Lemma 2 *If F is such that $BF \in \mathsf{Im}E^*$ and $A \preceq BF$ then F is solution for the problem (i.e. F is solution of Eq.9).*

Proof: If $BF \in \mathsf{Im}E^*$ then $BF = E^*BF$. As $A \preceq BF$, then

$$E^*(A \oplus BF)E^* = E^*BFE^* = BFE^*$$

and

$$(A \oplus BF)E^* = BFE^*$$

■

Definition 4 (Parallelism Relation (Katz, 2007)) *Let Z' and Z be matrices for a given idempotent semiring such that $Z' = Z \otimes m$ with m a given element of $\overline{\mathbb{Z}}_{\max}$. Therefore $Z' \sim Z$ and the relation $\sim$ is denoted as Parallelism Relation.*

Remark 3 *One can see that if Z is such $BZ \in \mathsf{Im}E^*$, then all Z' such that $Z' \sim Z$ is such that $BZ' \in \mathsf{Im}E^*$*

Proposition 3 *If the matrix B is G-astic and if there exists a matrix $Z \in \overline{\mathbb{Z}}_{max}^{p\times n}$ with non null entries such that $BZ \in \mathsf{Im}E^*$ there exists a solution for*

the proposed control problem (i.e. a solution of Eq.9). One solution is given by the least element of the set $\{Z'|(Z' \sim Z) \text{ and } (A \preceq BZ')\}$.
Proof:

The proof is given by constructing a solution for the Eq.9 from the assumptions of the theorem.

If Z *is such that* $(Z_{ij} \neq \varepsilon)$*, and* B *is G-astic, then by the lemma 1, it is always possible to make* $A \preceq BZ'$*, since one can make the matrix* $Z' \sim Z$ *as large as possible. In this sense, if we denote* $F = Z'$*, one can ensure that* $BF = E^*BF$ *and* $A \preceq BF$*. Therefore by the lemma 2,* F *is a solution for the proposed control problem. Obviously the least solution is given by the least element of the set* $\{Z'|(Z' \sim Z) \text{ and } (A \preceq BZ')\}$ ■

It is important to remark that the condition $BZ \in \mathsf{Im}E^*$ is equivalent to find Z such that:

$$BZ = E^*Y, \tag{19}$$

in which Y is some matrix of compatible dimension. To solve the equation $BZ = E^*Y$, one can use the algorithm presented in subsection 2.1. As informed in that subsection, if B and E^* have at least one finite element on each row and on each column, the presented algorithm converges if and only if a finite solution to this equation exists. These conditions regarding the matrices are completely fulfilled if B is G-astic, since by definition matrix B has at least one finite element in each column and matrix E^* has always at least one finite element on each row and on each column.

Corollary 1 *If* $B = I$*, that is the firing dates of the state transition are controllable, the presented control problem has always a solution.*
Proof:
The proof follows the one given for the theorem 3, by observing that always exists a matrix Z *with non null entries such that* $Z \in \mathsf{Im}E^*$*.* ■

Another result, concerning stability of the closed-loop system is given by the property 2.

Property 2 *If* B *is G-astic and the system is connected , the existence of feedback F,* $(F_{ij} \neq \varepsilon)$ *ensures that the closed-loop system is stable*[3] *.*
Proof:
The proof come from the fact that in this situation, the closed loop system is strongly connected and so stable. For more details see Commault (1998). ■

4 Application example: workshop control

The objective this section is to illustrate the application of the proposed approach to control a workshop subjected to some time restrictions. To this end, consider the TEG depicted in the Fig. 1 as an example.

It describes a model for a workshop with 3 machines (M_1, M_2 and M_3). Machines M_1 and M_2 can process parts which are assembled by machine M_3. Inputs u_1 and u_2 represent the admission dates of parts into the system and the transportation time are t_{u1} and t_{u2} for the respective inputs; machine M_1 can process one part in t_1 time units; machine M_2 can process one part in t_2 time units. The transportation time between machines M_1 and M_3 is t_{13} time units and between machines M_2

[3] Recall that a system is stable if the number of tokens in the places are always bounded.

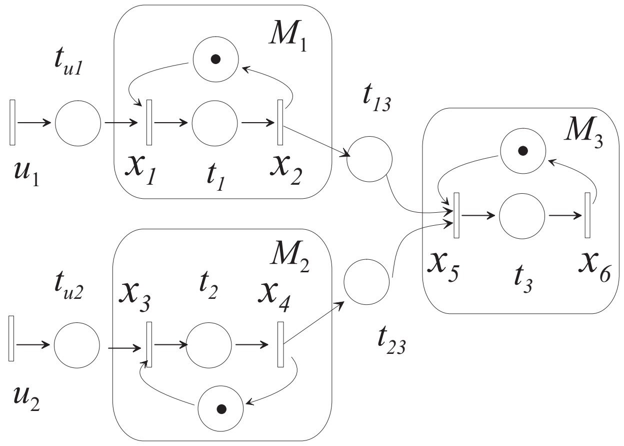

Fig. 1. Workshop System

and the one M_3 is t_{23} time units. Machine M_3 can assemble one part in t_3 time units.

For this example, the times are chosen as: $t_1 = 10$, $t_2 = 35$, $t_3 = 50$, $t_{13} = t_{23} = 2$, $t_{u1} = 6$ and $t_{u2} = 12$. One must observe that in this situation that machine M_1 has the greatest production rate, which is equal to $\frac{1}{10}$ and M_3 has the smallest, which is equal to $\frac{1}{50}$. One can also observe that the system can be unstable since the number of tokens in the places among M_1 and M_2 and M_3 can be unbounded. This is the case for the system impulse response, that is, when all the supply materials are available at date $t = 0$.

It can be shown that the maxplus model for this TEG is given by:

$$x(k) = A_0 x(k) \oplus A_1 x(k-1) \oplus B_0 u(k). \tag{20}$$

As a consequence, by remembering remark 1 (since A_0 has circuits with negative circuit weights) this equation can be rewritten as:

$$x(k) = A_0^* A_1 x(k-1) \oplus A_0^* B_0 u(k). \tag{21}$$

Therefore the matrix A for the system, by remembering the Eq. 1, is $A_0^* A_2$ and the matrix B is $A_0^* B_1$. At this point it is important to remark that the matrix B is G-astic.

In order to guarantee a desirable behavior for the system, it is necessary to define a set of operational restrictions. These restrictions, which lead to matrix E_r, are given below.

- The difference of dates, in which machines M_1 and M_2 deliver its products, must not exceed $r1$ time units, that is $x_2(k) - x_4(k) \leq r_1$ and $x_2(k) - x_4(k) \geq -r_1$
- The sojourn time of the materials into the system is limited, that is: $x_6(k) - x_1(k) \leq r_2$ and $x_6(k) - x_3(k) \leq r_3$.

Furthermore, in this case, by observing Eq. 20, one can see the state must also respect the inequality $A_0 x(k) \preceq x(k)$. As a consequence, the restriction matrix E is given by:

$$E = E_r \oplus A_0 \tag{22}$$

For the present application example, one chooses $r_1 = 0$ (that is, machines M_1 and M_2 must deliver its products at the same date) and the sojourn times as $r_2 = 65$ and $r_3 = 90$.

As remarked before, it can be checked that the matrix B is G-astic. In this sense, by using theorem 3, it was possible to find a feedback matrix for the system, which is given by:

$$F = \begin{bmatrix} 38 & 34 & 37 & 37 & 34 & 31 \\ 7 & 3 & 6 & 6 & 3 & 0 \end{bmatrix}$$

For instance, a possible firing date sequence for the system is:

$$x(0) = [22 \quad 35 \quad 0 \quad 35 \quad 37 \quad 87 \quad]^T,$$
$$x(1) = [124 \quad 134 \quad 99 \quad 134 \quad 136 \quad 186 \quad]^T,$$
$$x(2) = [223 \quad 233 \quad 198 \quad 233 \quad 235 \quad 285 \quad]^T.$$

One can easily see that the machines M_1 and M_2 deliver its products at the same date, as desired.

For the sake of comparison, it is presented below the results for the open-loop system (for which F is a null matrix):

$$x(0) = [22 \quad 35 \quad 0 \quad 35 \quad 37 \quad 87 \quad]^T,$$
$$x(1) = [35 \quad 45 \quad 35 \quad 70 \quad 87 \quad 137 \quad]^T,$$
$$x(2) = [45 \quad 55 \quad 70 \quad 105 \quad 137 \quad 187 \quad]^T.$$

It is clear that the machines M_1 and M_2 do not deliver its products at the same date.

Finally, it is important to remark that the obtained closed-loop system is strongly connect and therefore stable. In addiction, by using the maxplus toolbox of the Scilab J-P.Quadrat (2007), the cycle time for the system can be computed, and the result is 99 time units.

5 Conclusion

This paper has proposed a control methodology to design feedback controller for maxplus linear system subjected to state space restriction. The objective, in this case, was to design a controller that guarantees that the system evolves without violating timed restrictions imposed to the state. The presented approach is based uniquely on the algebraic property of the system matrices and the controller matrix is obtained by solving a maxplus linear equation. The main contribution was a sufficient condition that ensures the computation of the feedback. To illustrate the contribution of this paper a workshop problem was presented, for which a controller was designed in order to guarantee that closed-loop system respects some time restrictions.

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