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Constructive role of noise in signal detection from parallel arrays of quantizers

David Rousseau, François Chapeau-Blondeau*

Laboratoire d'Ingénierie des Systèmes Automatisés (LISA), Université d'Angers, 62 avenue Notre Dame du Lac, 49000 Angers, France

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Abstract

A noisy input signal is observed by means of a parallel array of one-bit threshold quantizers, in which all the quantizer outputs are added to produce the array output. This parsimonious signal representation is used to implement an optimal detection from the output of the array. Such conditions can be relevant for fast real-time processing in large-scale sensor networks. We demonstrate that, even for suprathreshold input signals, the presence of independent noises added to the thresholds in the array, can lead to a better performance in the optimal detection. We relate these results to the phenomenon of suprathreshold stochastic resonance, by which nonlinear transmission or processing of signals with arbitrary amplitude can be improved by added noises in arrays.

Keywords: Detection; Quantizer; Nonlinear arrays; Sensor arrays; Suprathreshold stochastic resonance

1. Introduction

When signal and noise are coupled nonlinearly, there exists a possibility for the noise to interact constructively with the signal, so that the presence of the noise can reveal beneficial. Stochastic resonance (SR) describes this possibility of a constructive action of the noise. Introduced some 20 years ago in the context of nonlinear physics, SR has progressively been reported in many areas, under many different forms, with various types of signals, nonlinear processes and measures of performance receiving improvement from the noise (see [1–5] for recent surveys). SR has also been applied specifically to standard signal processing problems, for instance to detection [6–9] or estimation [10].

So far, most studies have shown SR with nonlinear systems presenting thresholds or potential barriers, and in which the noise brings assistance to a small subthreshold signal in overcoming the nonlinearity for a more efficient response. Recently, another mechanism of improvement by noise was introduced under the

*Corresponding author. Fax: +33 2 41 73 53 52.
E-mail address: chapeau@univ-angers.fr
(F. Chapeau-Blondeau).

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name of suprathreshold SR [11]. This SR applies to threshold devices driven by a signal already above threshold which needs no assistance to overcome it. SR is thus absent in a single device, but appears when the devices are associated in a parallel array. Independent noises injected on the devices have the ability to increase the efficacy of representation of the signal by the array compared to a single device with no added noise. This translates into the possibility of improving various measures of performance, depending on the task at hand, by addition of noises in the array. This novel suprathreshold SR has been observed with the mutual information [11–13] and the input–output correlation [14] in random signal transmission, with a signal-to-noise ratio in periodic signal transmission [15], with the Fisher information in signal estimation [16].

In the present paper, we consider the same type of parallel arrays of comparators as used in previous suprathreshold SR studies [11–16], and we investigate them in the framework of an optimal detection task. We emphasize, as also done in the previous studies, that such arrays of nonlinearities bear similarities and significance to several areas including known technologies (like sonar arrays [17], flash analog-to-digital converters [18]), or promising avenues (like neural processing [19], cochlear implants [20], artificial vision [21], or other new-generation sensing devices [22]). The possibility of noise enhancement of information processing in nonlinear arrays, with its various modalities, is thus a new property with rich potentialities to be explored, an endeavor to which the present study participates.

2. Optimal detection from the output of a nonlinear parallel array

We consider a detection task where an input signal $s(t)$ can be one of two known signals, $s(t) \equiv s_0(t)$ with prior probability P_0, or $s(t) \equiv s_1(t)$ with prior probability $P_1 = 1 - P_0$. The input signal $s(t)$ is buried in an input noise $\xi(t)$ with probability density function $f_\xi(u)$. This yields the input signal–noise mixture $s(t) + \xi(t) = x(t)$. This mixture $x(t)$ is observed by means of a parallel array of N threshold comparators or one-bit quantizers, following the setting of [11–13]. We arrange for the possibility of a noise $\eta_i(t)$, independent of $x(t)$, to be added to $x(t)$ before quantization by quantizer i. Quantizer i, with threshold θ_i, delivers the output

$$y_i(t) = U[x(t) + \eta_i(t) - \theta_i], \quad i = 1, 2, \ldots N, \tag{1}$$

where $U(u)$ is the Heaviside function, i.e. $U(u) = 1$ if $u > 0$ and is zero otherwise. The response $Y(t)$ of the array is obtained by summing the outputs of all the quantizers, as

$$Y(t) = \sum_{i=1}^{N} y_i(t). \tag{2}$$

The array output $Y(t)$ of Eq. (2) is measured at M distinct times t_k, for $k = 1$ to M, so as to provide M data points $Y_k = Y(t_k)$. Each one of the Y_k can assume only $N + 1$ discrete integer values from 0 to N, so a total of $(N + 1)^M$ discrete states are accessible to the data $\boldsymbol{Y} = (Y_1, \ldots, Y_M)$. We then want to use the data $\boldsymbol{Y}$ to decide whether the noisy input $x(t)$ is formed by $\xi(t)$ mixed to $s_0(t)$ (hypothesis H_0) or to $s_1(t)$ (hypothesis H_1).

According to classical detection theory [23], the detector that minimizes the overall probability of detection error P_{er}, uses the likelihood ratio $L(\boldsymbol{Y})$ to implement the test

$$L(\boldsymbol{Y}) = \frac{\Pr\{\boldsymbol{Y}|\mathrm{H}_1\}}{\Pr\{\boldsymbol{Y}|\mathrm{H}_0\}} \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \frac{P_0}{P_1}, \tag{3}$$

and in doing so achieves the minimal P_{er} expressable as

$$P_{\mathrm{er}} = \frac{1}{2} - \frac{1}{2}\sum_{\boldsymbol{Y}} |P_1 \Pr\{\boldsymbol{Y}|\mathrm{H}_1\} - P_0 \Pr\{\boldsymbol{Y}|\mathrm{H}_0\}|, \tag{4}$$

where the sum in Eq. (4) runs over the $(N + 1)^M$ states accessible to the data $\boldsymbol{Y}$.

We will consider here that the N threshold noises $\eta_i(t)$ are white (strict sense), mutually independent, and identically distributed with cumulative distribution function $F_\eta(u)$ and probability density function $f_\eta(u) = \mathrm{d}F_\eta/\mathrm{d}u$. We also consider that the input noise $\xi(t)$ is white, just as the threshold noises $\eta_i(t)$ are. The conditional probabilities therefore factorize as $\Pr\{\mathbf{Y}|\mathrm{H}_j\} = \prod_{k=1}^{M} \Pr\{Y_k|\mathrm{H}_j\}$, for $j \in \{0,1\}$.

At any time t, for a given fixed value x of the input signal $x(t)$, we have, according to Eq. (1), the conditional probability $\Pr\{y_i(t)=0|x\}$ which is also $\Pr\{x+\eta_i(t)\leqslant\theta_i\}$, this amounting to $\Pr\{y_i(t)=0|x\} = F_\eta(\theta_i - x)$. In the same way $\Pr\{y_i(t)=1|x\} = 1 - F_\eta(\theta_i - x)$.

We assume for the present time, as done in [12,14], that all the thresholds θ_i share the same value $\theta_i = \theta$ for all i. The conditional probability $\Pr\{Y(t)=n|x\}$ then follows, according to the binomial distribution [24], as

$$\Pr\{Y(t)=n|x\} = C_n^N[1-F_\eta(\theta-x)]^n \times F_\eta(\theta-x)^{N-n}, \quad (5)$$

where C_n^N is the binomial coefficient. Since $x(t) = s_j(t) + \xi(t)$, with $j=0$ or 1, the probability density for the value x is $f_\xi(x - s_j(t))$. We therefore obtain the probability

$$\Pr\{Y(t)=n|\mathrm{H}_j\} = \int_{-\infty}^{+\infty} C_n^N\,[1-F_\eta(\theta-x)]^n \times F_\eta(\theta-x)^{N-n} f_\xi(x-s_j(t))\,\mathrm{d}x. \quad (6)$$

Eq. (6) now allows an explicit evaluation of the optimal detector of Eq. (3) and of its performance P_{er} of Eq. (4).

3. Constructive action of the threshold noises

We proceed first with the simple case where the signals to be detected are constant signals $s_0(t) \equiv s_0$ and $s_1(t) \equiv s_1$, with two known constants $s_0 \neq s_1$. In this case, Fig. 1 illustrates the possibility of improving the detection performance measured by P_{er} of Eq. (4) in the presence of the threshold noises $\eta_i(t)$.

Fig. 1 shows that application of the threshold noises $\eta_i(t)$ in the array leads to a reduction of the probability of error P_{er} in the optimal detection task at the output, with an optimal nonzero amount of the threshold noises $\eta_i(t)$ where P_{er} is minimized. The effect is not present with a single quantizer ($N=1$) and it gets more pronounced as N increases. Qualitatively, this effect can be related to a richer representation capability at the array output where the presence of the threshold noises $\eta_i(t)$ allows the quantizers to respond differently, as opposed to the case with no threshold noises where all the quantizers respond in unison just like a single quantizer with a poorer representation capability. Suprathreshold SR, as observed in [11–16], was also based on such constructive action of the threshold noises in arrays, although the tasks at hand and the measures of performance were distinct. The treatment of Section 2 exemplified by Fig. 1, reveals and describes in a quantitative way that a constructive action of the threshold noises can also occur in an optimal detection task from the array output. A validation by a Monte Carlo simulation of the optimal detector of Eq. (3) is also offered in Fig. 1. The complexity of implementing this detector is linear with the array size N since it is based on the sum of the N quantizer outputs.

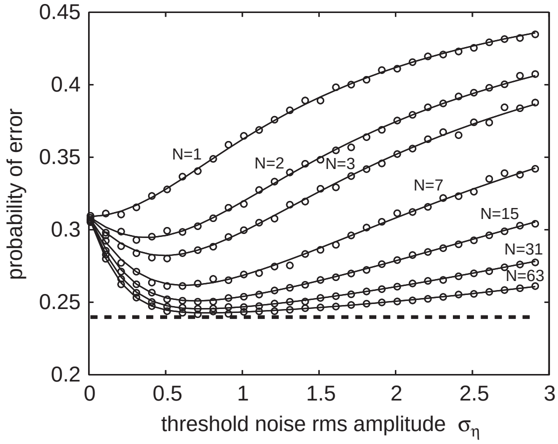

Fig. 1. Probability of error P_{er} of the optimal detector from the array output, as a function of the rms amplitude σ_η of the threshold noises $\eta_i(t)$ chosen zero-mean Gaussian. The input noise $\xi(t)$ is zero-mean Gaussian with rms amplitude $\sigma_\xi = 1$. The signals to be detected are $s_0(t) \equiv s_0 = 0$ and $s_1(t) \equiv s_1 = 1$. All the thresholds in the array are set to $\theta = (s_0+s_1)/2 = 1/2$. Also $P_0 = 1 - P_1 = 1/2$, and $M = 2$. The solid lines are from the theory of Eq. (4). The sets of discrete points (○) are from a Monte Carlo simulation of the optimal detector from Eq. (3). The dashed line is the probability of error Eq. (7) of the same optimal (minimal P_{er}) detector operating directly on the input signal–noise mixture $x(t)$.

For comparison of the optimal detector from the array ouput that is addressed in Fig. 1, it is interesting to consider the same optimal detector (the minimum P_{er} detector) that would operate directly on the input signal–noise mixture $x(t)$ rather than on its quantized representation by the array. We shall write $P_{\mathrm{er}}^{\mathrm{in}}$ the probability of error of this detector, which is also given, according to classical detection theory [23], by an expression formally similar to Eq. (4), and which reduces, in the case of a Gaussian input noise $\xi(t)$, to

$$P_{\mathrm{er}}^{\mathrm{in}} = \frac{1}{2}\left[1 + P_1 \,\mathrm{erf}\left(\sqrt{M}\,\frac{x_T - s_1}{\sqrt{2}\sigma_\xi}\right) - P_0 \,\mathrm{erf}\left(\sqrt{M}\,\frac{x_T - s_0}{\sqrt{2}\sigma_\xi}\right)\right]. \quad (7)$$

In Fig. 1, the probability of error $P_{\mathrm{er}}^{\mathrm{in}}$ of Eq. (7) is represented by the dashed line. It can be noted in Fig. 1 that, in the optimal detection from the array output, the minimal value reached by P_{er} at the optimal level of the threshold noises, as N increases, tends to the performance $P_{\mathrm{er}}^{\mathrm{in}}$ of the optimal detector directly applied on the input signal–noise mixture $x(t)$. This proves that the optimal detector from the output of the array of one-bit quantizers, as the array size becomes large, is able to perform as efficiently as the optimal detector operating directly on the analog input signal $x(t)$. At the same time, Fig. 1 shows that with relatively modest sizes N, the performance of the array comes close to the best performance $P_{\mathrm{er}}^{\mathrm{in}}$ at the input. Advantages afforded by the array lie in the parsimony of the representation and simplicity of operation (possibly associated to rapidity), working on a few bits collected by the comparators, as opposed to the infinite number of bits in principle associated with the analog input $x(t)$. Such conditions can be specially relevant to fast real-time processing in large-scale arrays of low-complexity low-cost sensors, possibly with neural inspiration.

The observation in Fig. 1 that the detection from the ouput $Y(t)$ of the array of quantizers can reach a performance as good as the detection from the input signal–noise mixture $x(t)$, is reminiscent of a linearizing action of the noise that is evoked in the dithering phenomenon with quantizers [25]. Dithering can be described as a linearization of the threshold characteristic of a quantizer, caused by an added noise uniform over the quantization step. Dithering can be seen as one possible form of improvement by noise, with specific nonlinearities (quantizers) and a specific measure of performance (a linearized characteristic). If other measures of performance are to be maximized, like for instance a signal-to-noise ratio for periodic signal transmission, other optimum conditions for the added noise come out [26]. The probability of detection error P_{er} is yet another measure of performance, which is here meaningful for the detection with arrays of quantizers. The behavior of P_{er} in Fig. 1 as noise is added, although it displays some aspects reminiscent of dithering, cannot be completely deduced from consideration of dithering, and it needs to be specifically studied. Improvements by noise in nonlinear processes can occur which are markedly distinct from a linearization effect such as dithering. It is known that quantizers, isolated [26] or assembled in parallel arrays [15], are capable of realizing input–output gains in the signal-to-noise ratio of a sinewave in noise, which can be made larger than unity thanks to added noise. Linearized systems as produced by dithering can at best achieve a unit gain, but not a gain amplification above unity. This legitimates explicit inspection of the impact of noise in nonlinear processes with their appropriate measures of performance, each time new potentially useful configurations are uncovered, as it is the case here with parallel arrays of quantizers for signal detection.

4. Influence of the different parameters

By application of the theory of Section 2, we have verified that the constructive action of the threshold noises $\eta_i(t)$ on the detection performance, as illustrated by Fig. 1, is robustly preserved in a broad range of conditions. This is especially true upon changes of the probability densities $f_\xi(u)$ and $f_\eta(u)$, of the location of the common threshold θ, and of P_0. In each case, the quantitative details of the effect can be worked out with the theory of Section 2. We choose next to focus on the influence of the number M of measurements. Later on, Figs. 4 and 5 show that the effect is also qualitatively preserved with a uniform density $f_\xi(u)$ at the input.

Concerning the number M of measurements, a constructive action of the threshold noises $\eta_i(t)$ is observed for any value of M (see Fig. 2A), except for $M = 1$, as visible in Fig. 2B. An explanation for this point can be given in this way. When $M = 1$, in the absence of the threshold noises $\eta_i(t)$, the detector from the array output and the detector from the input $x(t)$ operate in an equivalent way (see also Section 6): they both compare their single measurement, respectively $Y(t_1) = NU[x(t_1) - (s_0 + s_1)/2]$ and $x(t_1)$, to their detection threshold according to Eq. (3). This mode of operation leads to the same P_{er} at $\sigma_\eta = 0$, as shown by Fig. 2B , and to a P_{er} which degrades when the threshold noises $\eta_i(t)$ are added. It is at $M > 1$ that the detections from the input and from the output, cease to be comparable as two decisions taken from a single measurement $x(t_1)$, either directly or after quantization. At $M > 1$, each detector has to collect in an optimal way its M data points, so as to produce the single scalar value on which the detection will be based. This way of collecting the data is in general a nonlinear combination, which is performed in a specific way at the output of the array. What the present study demonstrates then, and which is by no means a priori obvious, is that this nonlinear combination allows some room where the action of the threshold noises can result in improved detection performance.

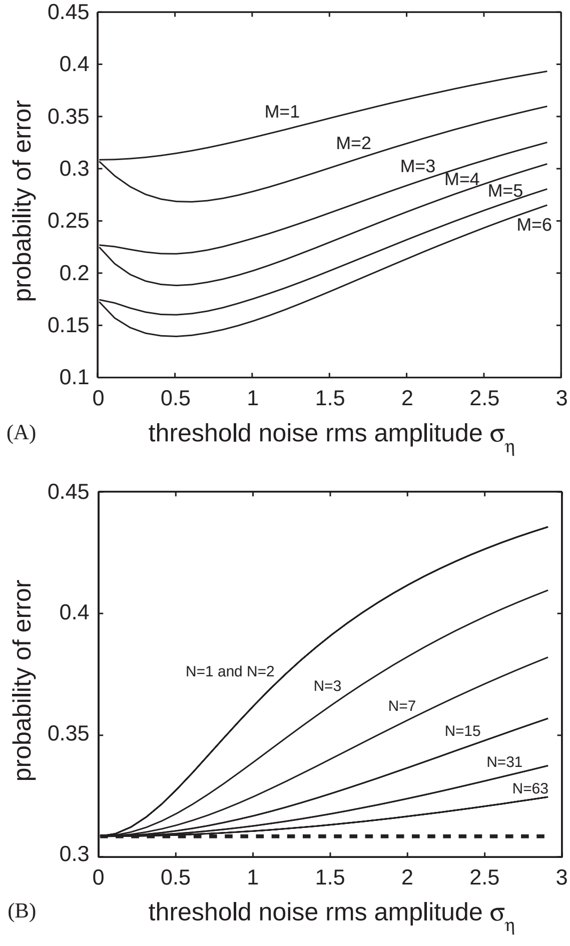

Fig. 2. Influence of the number M of measurements. Panel A: same conditions as Fig. 1 except a fixed array size $N = 5$. Panel B: same conditions as Fig. 1 except $M = 1$.

5. Distribution of the quantization thresholds

We shall now examine the case where the thresholds θ_i no longer share the same value θ. This case where the quantization thresholds θ_i can be separately adjusted corresponds a priori to a more efficient configuration of the array of N comparators. This is what is done, for instance, in flash analog-to-digital converters. When the thresholds θ_i, for $i = 1$ to N, no longer share the same value θ, the conditional probability $\Pr\{Y(t) = n|x\}$ of Eq. (5) has to be computed as

$$\Pr\{Y(t) = n|x\} = \sum_{(n)} \prod_{i=1}^{N} [1 - F_\eta(\theta_i - x)]^{y_i} F_\eta(\theta_i - x)^{1-y_i}, \qquad (8)$$

where $\sum_{(n)}$ stands for the sum over the C_n^N configurations accessible to the N comparators for which the number of y_i equal to 1 is exactly n, among the 2^N distinct configurations accessible to the N comparators. After this replacement of Eq. (5) by Eq. (8) is done, the probability $\Pr\{Y(t) = n|\mathrm{H}_j\}$ for $j \in \{0, 1\}$, follows in the same way as

$$\Pr\{Y(t) = n|\mathrm{H}_j\} = \int_{-\infty}^{+\infty} \Pr\{Y(t) = n|x\} f_\xi(x - s_j(t))\, \mathrm{d}x. \qquad (9)$$

Knowledge of $\Pr\{Y(t)=n|\mathrm{H}_j\}$ from Eq. (9) again allows an explicit evaluation of the optimal detector of Eq. (3) and of its performance P_{er} of Eq. (4).

To proceed, some criterion has to be introduced to specify the distribution of the thresholds θ_i. If an optimal distribution is sought for the thresholds, usually this distribution will be specific to a given criterion, and it will depend upon the signals to be detected, upon the types of the input and threshold noises, and upon the size N of the array. In definite conditions, this optimization problem may be uneasy to solve. For constant signals $s_0(t)\equiv s_0$ and $s_1(t)\equiv s_1$, a reasonable choice is a regular distribution with the N thresholds θ_i evenly covering the interval between s_0 and s_1 via

$$\theta_i = s_0 + i\,\frac{s_1-s_0}{N+1}, \quad i=1,2,\ldots N. \tag{10}$$

This is the simple choice that is implemented by flash analog-to-digital converters.

We select this simple distribution of Eq. (10), to illustrate in Fig. 3 that a constructive action of the threshold noises $\eta_i(t)$ on the detection performance is still possible with distributed thresholds, in definite conditions. In Fig. 3, for all array sizes N, the input signal $s(t)$ is always suprathreshold, in

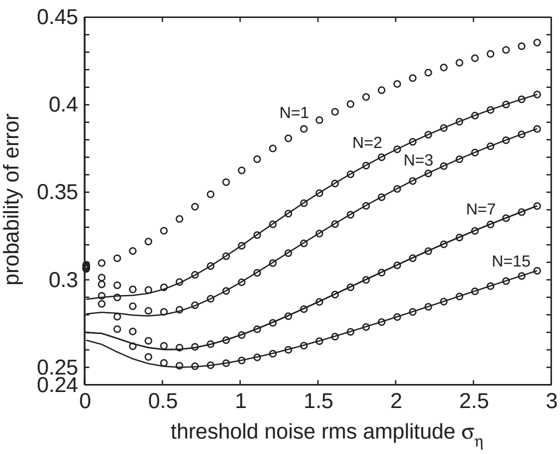

Fig. 3. Influence of the threshold distribution. The sets of discrete points (○) are obtained in the same conditions as Fig. 1 with a single common threshold $\theta=(s_0+s_1)/2=1/2$. The solid lines are also obtained in the same conditions as Fig. 1 except for the thresholds θ_i which are distributed according to Eq. (10).

the sense that, without assistance from the threshold noises $\eta_i(t)$, $s(t)$ alone when it switches from s_0 to s_1, is capable to induce transitions in any one of each quantizer ouput $y_i(t)$. As visible in Fig. 3, when no threshold noises $\eta_i(t)$ are present, the optimal detection realized by the array with distributed thresholds is more efficient than that with a single common threshold θ. At $\sigma_\eta=0$, the probability of error P_{er} decreases as the array size N increases; as expected, the threshold distribution enhances the efficacy of the detection. Still, even in this situation, the detection performance can be further improved by the action of the threshold noises $\eta_i(t)$. For sufficiently large array size N (starting at $N=3$ in Fig. 3), when the threshold noises $\eta_i(t)$ are added, the probability of error P_{er} experiences a nonmonotonic evolution as the threshold noise rms amplitude σ_η is increased. This demonstrates that a constructive action of the threshold noises $\eta_i(t)$ can take place with distributed thresholds, as well as with a single common threshold, in the detection from the array output.

Furthermore, it is interesting to notice in Fig. 3, that at the optimal level of the threshold noises σ_η, the distributed-threshold array does not perform better than the common-threshold array. The added threshold noises equalize the performance of both configurations. In the face of this outcome, a noticeable superiority can be assigned to the common-threshold configuration of the array, in terms of simplicity. This configuration requires, to set up the common threshold θ, only the prior knowledge of the mean value between the two signals to be detected, i.e. $(s_0+s_1)/2$. For instance, it suffices to know that the two signals to be detected are formed by a bipolar waveform $\pm A$, with possibly unknown A, to optimally set up the common threshold at $\theta=0$. By contrast, the distributed-threshold configuration of the array requires the prior knowledge of both signal values s_0 and s_1, or the knowledge of A with the bipolar signals, so as to distribute the thresholds in the range between s_0 and s_1. Fig. 3 shows that the same performance will be reached by these two array configurations, at the optimum of the threshold noises.

6. Linear detection at the array output

The likelihood ratio test of Eq. (3) is the optimal (minimal P_{er}) detector based on the complete data set $\boldsymbol{Y} = (Y_1, \ldots Y_M)$ collected at the output of the array. This optimal test of Eq. (3), as visible from the derivation of Section 2, generally computes a nonlinear expression of the measurements Y_k, $k = 1, \ldots M$, to serve as a test statistic. For a matter of simplicity, it can be relevant to impose to base the detection, not on the complete vector data $\boldsymbol{Y}$, but only on its sample mean $M^{-1}\sum_{k=1}^{M} Y_k$ or equivalently its sample sum $\sum_{k=1}^{M} Y_k = Y_{\mathrm{cum}}$. The same principles of detection theory [23] express that the optimal (minimal P_{er}) detector based on Y_{cum} is again a test of the likelihood ratio, this time taking the form

$$L(Y_{\mathrm{cum}}) = \frac{\Pr\{Y_{\mathrm{cum}}|\mathrm{H}_1\}}{\Pr\{Y_{\mathrm{cum}}|\mathrm{H}_0\}} \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \frac{P_0}{P_1} . \tag{11}$$

To express the likelihood ratio $L(Y_{\mathrm{cum}})$ of Eq. (11), the probabilities $\Pr\{Y_{\mathrm{cum}} = K|\mathrm{H}_j\}$ for $K = 0$ to $M \times N$, are computable from the probabilities in Eq. (6) of the individual components Y_k, by monitoring the different ways these Y_k sum up in Y_{cum} to a given K. This will in general also lead, as seen from the form of Eq. (6), to a nonlinear expression, of Y_{cum} this time, serving as a test statistic. It is possible to take the simplification further, by imposing for the detection a linear test of the sample sum Y_{cum} under the form

$$Y_{\mathrm{cum}} \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \gamma . \tag{12}$$

With this imposed linear structure to the detector, the detection threshold γ will be selected so as to minimize the probability of error

$$P_{\mathrm{er}} = P_0 \Pr\{Y_{\mathrm{cum}} > \gamma|\mathrm{H}_0\} + P_1 \Pr\{Y_{\mathrm{cum}} < \gamma|\mathrm{H}_1\} . \tag{13}$$

Expressing this P_{er} of Eq. (13) for minimization will again necessitate the probabilities $\Pr\{Y_{\mathrm{cum}} = K|\mathrm{H}_j\}$ for $K = 0$ to $M \times N$, computable from the probabilities in Eq. (6). The resulting detector based on Y_{cum} and Eq. (12) is in principle a suboptimal detector not as efficient as the optimal detector based on $\boldsymbol{Y}$ and Eq. (3). On the other hand, the detector based on Y_{cum} and Eq. (12) can be viewed as a simpler (linear) detector compared to the generally nonlinear detector based on $\boldsymbol{Y}$ and Eq. (3). For comparison, Fig. 4 shows the performances of both detectors in representative conditions, and their evolutions when the level σ_η of the threshold noises $\eta_i(t)$ is raised.

As expected, in Fig. 4 in general the performance of the suboptimal detector from Y_{cum} and Eq. (12) is not as good as that of the optimal detector from $\boldsymbol{Y}$ and Eq. (3). However, at zero threshold noises $\sigma_\eta = 0$, both performances coincide. This is because in this case the N quantizers switch in unison, just as if there were only one quantizer, and the optimal test of Eq. (3) reduces to an equivalent test under the form of Eq. (12). This equivalence between Eq. (3) and (12) breaks down when the threshold noises $\eta_i(t)$ are added, and expectedly optimal Eq. (3) outperforms suboptimal Eq. (12) in Fig. 4. The two performances tend to rejoin again in the limit of large levels of the threshold noises. The interesting feature revealed by Fig. 4 is that both detectors,

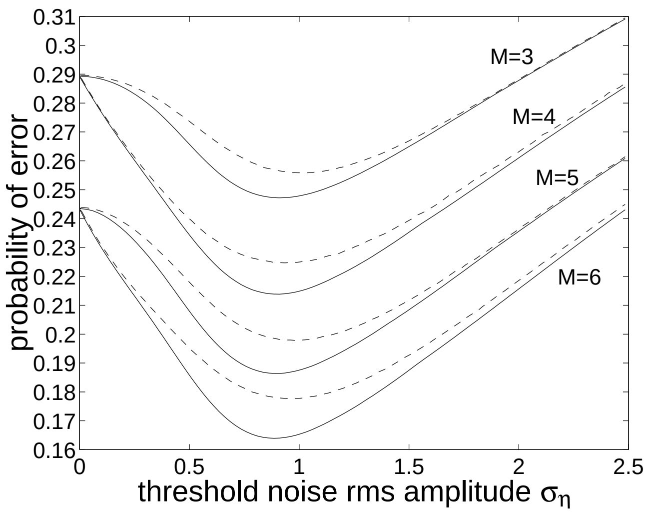

Fig. 4. Probability of error P_{er}, as a function of the rms amplitude σ_η of the zero-mean Gaussian threshold noises $\eta_i(t)$, for $s_0(t) \equiv s_0 = 0$ and $s_1(t) \equiv s_1 = 1$ in zero-mean uniform noise $\xi(t)$ of rms amplitude $\sigma_\xi = 1$. Solid lines: optimal detector from $\boldsymbol{Y}$ and Eq. (3), dashed lines: linear suboptimal detector from Y_{cum} and Eq. (12). Also $P_0 = 1/2$, $N = 5$, and $\theta_i = 0.5, \forall i$.

from $\boldsymbol{Y}$ and from Y_{cum}, benefit from the presence of a nonzero amount of the threshold noises $\eta_i(t)$. Although the specific values of the optimal noise level may differ in each case, the qualitative possibility of a constructive action of the threshold noises is preserved. More detailed analysis in this direction remain open for further investigation.

7. Application to sinewave detection

Eq. (6) allows the implementation of the optimal detector of Eq. (3) for the detection of any two known signals $s_j(t)$, $j \in \{0, 1\}$. We tested above the case of two constant signals, essentially as the simplest configuration to exhibit and analyze the possibility of a constructive action of the threshold noises $\eta_i(t)$ in the detection. From a practical standpoint, it can also be interesting to consider the detection of a known sinusoidal signal $s_1(t) = \cos(2\pi t/T_s)$ in noise (i.e. $s_0(t) \equiv 0$). In this case, application of Eq. (6) directly specifies the optimal detector of Eq. (3). Its performance P_{er} of Eq. (4) is shown in Fig. 5, for $M = 6$ data points $Y(t_k)$ sampled at times $t_k = (k-1)T_s/M$, $k = 1$ to 6. Without going into a detailed analysis, the main point we want to emphasize in Fig. 5 is that the possibility of a beneficial action of the threshold noises $\eta_i(t)$ on the performance is preserved in the detection of a known sinewave from the array output.

Another case of practical interest is the detection of a sinewave $s_1(t) = A\cos(2\pi t/T_s + \phi)$ with unknown parameters, for instance an unknown phase ϕ (an unknown amplitude A or both could be treated equivalently), and $s_0(t) \equiv 0$. Essentially two approaches are followed in classical detection theory [23]. One is to treat ϕ (or A, or both) as a deterministic parameter with a fixed unknown value which is estimated by maximum likelihood as $\widehat{\phi} = \text{argmax} \Pr\{\boldsymbol{Y}|\phi; \text{H}_1\}$, with $\Pr\{\boldsymbol{Y}|\phi; \text{H}_1\}$ which is known through Eq. (6), and then implement the (generalized likelihood ratio) test of Eq. (3) where this time $L(\boldsymbol{Y}, \widehat{\phi}) = \Pr\{\boldsymbol{Y}|\widehat{\phi}; \text{H}_1\}/\Pr\{\boldsymbol{Y}|\text{H}_0\}$. Generally there is no optimality of this approach, especially in terms of P_{er}, only asymptotically. The other approach is to treat ϕ (or A, or both) as a random parameter

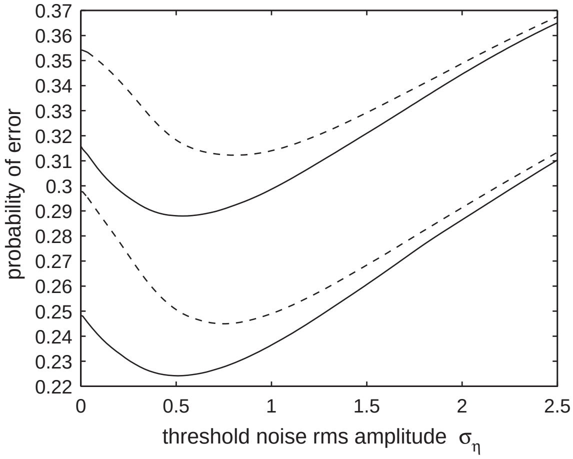

Fig. 5. Probability of error P_{er} of the optimal detector from the array output, as a function of the rms amplitude σ_η of the threshold noises $\eta_i(t)$ chosen zero-mean Gaussian. The input noise $\xi(t)$ is Gaussian (solid lines) or uniform (dashed lines) with zero-mean and rms amplitude $\sigma_\xi = 1$. The signal to be detected is a sinewave $s_1(t) = \cos(2\pi t/T_s + \phi)$ with known phase (two lower curves) or unknown phase (two upper curves). Also $P_0 = 1/2$, $N = 5$, $M = 6$ and $\theta_i = 0, \forall i$.

with a known probability density $p_\phi(\phi)$. For a given realization of ϕ, the probability $\Pr\{Y(t) = n|\phi; \text{H}_1\}$ is obtained by inserting the sinusoidal expression of $s_1(t)$ in Eq. (6). Then, the probability$\Pr\{\boldsymbol{Y}|\text{H}_1\} = \int_\phi \prod_{k=1}^{M} \Pr\{Y_k|\phi; \text{H}_1\} p_\phi(\phi)\, d\phi$ is used to apply the test of Eq. (3). This approach is optimal in the sense that it minimizes the overall probability of detection error P_{er} when computed with the averages taken both over the noise realizations and over the random phase. The performance P_{er} of this approach, resulting from Eq. (4), is also shown in Fig. 5, when ϕ is uniform over $[0, 2\pi)$. Again, without going into details, our main point in Fig. 5 is that the possibility of a beneficial action of the threshold noises $\eta_i(t)$ on the performance is preserved in the detection of a sinewave with unknown phase from the array output.

8. Discussion

The constructive action of the threshold noises $\eta_i(t)$ in the detection from the array output that we have shown possible here, is very similar, at least qualitatively, to the effect of suprathreshold SR as introduced in [11] and further developed in [12–16]. The present results on detection can even be interpreted as another further extension of suprathreshold SR. A noticeable difference though, is that here we investigated the effect of noise on the performance of *the* optimal detector from the array output, and by definition this detector is noise-dependent (beneficially, we showed). By contrast, previous studies on suprathreshold SR [11–16] dealt with a fixed "hard-wired" processing system. In spite of this distinction, it is interesting at another level to realize the marked common feature shared by these phenomena: the possibility of improving the processing at the ouput of parallel arrays of nonlinearities, for signals with arbitrary amplitudes, thanks to the action of independent noises injected into the array.

We have shown here essentially the feasibility in principle of this phenomenon in a detection task. In this context, many aspects remain open for further investigations, which are accessible based on the framework of Section 2. Other detection strategies, like Neyman–Pearson or Bayesian-cost, could be tested, and it is likely that the effect will persist in these conditions, this remaining to be studied explicitly. Still other extensions of the effect, beyond detection and the other previous forms [11–16], can also be sought, to complement and appreciate all the capabilities of information processing aided by noise with parallel arrays of nonlinearities, and progressively elaborate a unified picture.

Beyond the specific detection task considered here, the present results can carry relevance for several broader areas. First, they add another step in the on-going inventory and analysis of the different modalities of improvement by noise in nonlinear processes, in the line of SR. Second, the present results, together with previous results on suprathreshold SR in arrays, could lead to useful applications for real-time processing in existing or future multisensor networks or distributed intelligent systems. Third, the present results can also be meaningful for nonlinear processing by arrays of sensory neurons. Neurons are natural devices with intrinsic threshold nonlinearities, organized in networks, operating in noisy conditions (of external or internal origins), and achieving high efficiency for signal processing. Noise in these arrays may play a part to contribute to the performance, through detailed mechanisms which largely remain to be elucidated, and which may entail useful novel applications.

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