

# An experimental study of acoustoelectric transducers with nonuniform distribution of the piezoelectric coefficient

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(Received 30 August 1988; accepted for publication 30 June 1989)

In this paper, an experimental test of a theoretical model published previously is presented that describes the behavior of an acoustoelectric transducer with a nonuniform distribution of the piezoelectric coefficient within its bulk. Results of this theoretical model are first reviewed. Uniform and nonuniform piezoelectric transducers were fabricated, following a procedure described herein. The receive transfer functions of the transducers were recorded experimentally, and a comparison is made with the theoretical transfer functions predicted by the model, which shows good agreement. The transmit transfer functions of the uniform and nonuniform transducers were also measured and are reported. Numerical calculations of the different transfer functions given by the theoretical model for a uniform transducer associated with different backing materials are also presented, and the results are shown to be equivalent to the results following from the Mason equivalent circuit. Comparisons with experimental results and with Mason's equivalent circuit verified the new theoretical model.

PACS numbers: 43.88.Ar, 43.88.Fx, 43.35.Yb

## INTRODUCTION

In a previous paper,<sup>1</sup> we developed a theoretical model describing the behavior of a piezoelectric transducer with a nonuniform distribution of the piezoelectric coefficient within its bulk, when submitted to an arbitrary distribution of acoustic pressure. In the framework of this model, an expression for the receive transfer function of such a nonuniform transducer was derived. This theoretical model, on the one hand, provides a ready means of relating explicitly the transfer function of a nonuniform transducer to the distribution of the piezoelectric coefficient; but, on the other hand, more widely, it offers an alternative way, besides the equivalent circuit approaches, of describing theoretically, in quite general terms, the behavior of piezoelectric transducers in the receiving mode.

In the present paper, we propose an experimental test of the theoretical model. Several transducers, with uniform or nonuniform distribution of the piezoelectric coefficient, have been fabricated. The receive transfer functions of these transducers were measured experimentally, and the results are compared to the theoretical predictions of the model. The transmit transfer functions of the transducers were also recorded and are presented. The influence of the backing medium of a transducer on its receive transfer function, and the ability of the model to describe it, was also examined. For this purpose, a numerical study has been conducted in which the different transfer functions predicted by the model for a transducer with different backing materials, were compared to the transfer functions that follow from the application of the Mason equivalent circuit of the transducer.

In the course of this paper, we shall first review some aspects of the theoretical model, then the results of the ex-

perimental characterization of the transducers will be presented and discussed, followed by the numerical study of the backing materials.

## I. THEORETICAL BASIS

We consider an acoustoelectric transducer, depicted in Fig. 1, made of a plate of piezoelectric material of thickness  $e$ , and electroded on both faces. An axis  $Oz$  defines the direction perpendicular to the plane of the plate, and we shall employ a one-dimensional description of the system, assuming that the spatial variations of the quantities involved occur only in the  $Oz$  direction.

The front electrode of the transducer, located at abscissa  $z = 0$ , is in contact with a propagating medium of specific acoustic impedance  $Z_1$ , is taken as the reference for electric potentials. The rear electrode of the transducer, located at abscissa  $z = e$ , is in contact with a backing medium of specific acoustic impedance  $Z_3$ . The piezoelectric material constituting the transducer has a specific acoustic impedance  $Z_2$ ,

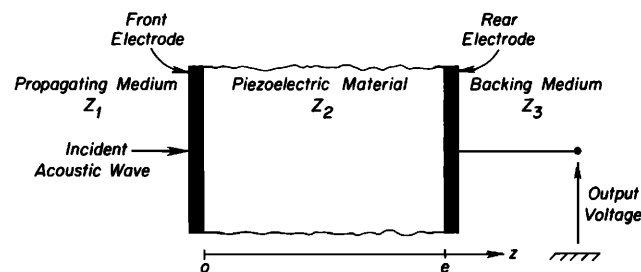


FIG. 1. Configuration of the piezoelectric transducer.

and its piezoelectric coefficient  $g_{33}$  is allowed to be nonuniform in the  $Oz$  direction and is represented by the function  $g_{33}(z)$ .

An acoustic plane wave traveling along  $Oz$  and impinging on the front face at  $z = 0$  drives the transducer in the thickness direction. In these conditions, it is possible to define, in the frequency domain, a receive transfer function  $T(\omega)$  as the ratio of the spectrum of the open circuit voltage measured on the rear electrode at  $z = e$ , to the spectrum of the acoustic pressure impinging on the front face of the transducer. We have demonstrated in Ref. 1 that this transfer function can be expressed as

$$T(\omega) = \tau_{12} \int_0^e g_{33}(z) A(z, \omega) dz, \quad (1)$$

where  $A(z, \omega)$  is a complex function defined as

$$A(z, \omega) = \frac{\exp[i\gamma(e-z)] + r_{23} \exp[-i\gamma(e-z)]}{\exp(i\gamma e) + r_{12} r_{23} \exp(-i\gamma e)}, \quad (2)$$

and with  $\tau_{12} = 2Z_2/(Z_2 + Z_1)$ ,  $r_{12} = (Z_2 - Z_1)/(Z_2 + Z_1)$  and  $r_{23} = (Z_3 - Z_2)/(Z_3 + Z_2)$ . The parameter  $\gamma$  is the propagation constant of acoustic waves in the transducer; it is a complex function of the angular frequency  $\omega$ , defined as

$$\gamma(\omega) = \beta(\omega) - i\alpha(\omega), \quad (3)$$

where  $\alpha(\omega)$  is a real function representing the attenuation of acoustic waves in the transducer, and  $\beta(\omega) = \omega/c$ ,  $c$  being the phase velocity of acoustic plane waves in the transducer.

So Eq. (1), together with Eq. (2), provides a theoretical model describing the receive transfer function of a piezoelectric transducer, in the general case where the piezoelectric coefficient  $g_{33}$  of the material is nonuniform throughout the thickness. It also includes the influence of the backing medium.

In the following, we present different tests of the predictions of this theoretical model in different situations.

## II. EXPERIMENTAL CHARACTERIZATION OF TRANSDUCERS

### A. Fabrication of the transducers

Transducers with nonuniform distributions of the piezoelectric coefficient have been fabricated with two different piezoelectric materials: a lead-zirconate titanate (PZT) ceramic and a polymer, polyvinylidene fluoride (PVDF).

To implement physically the nonuniform distributions  $g_{33}(z)$ , the transducers have been given a multilayered structure. Each layer of material, through a separate poling, can receive a specified value for its piezoelectric coefficient. Reversing the poling direction of a layer permits a change in the sign of its piezoelectric coefficient. Several layers of this type, stacked together, provide the definition of the distribution  $g_{33}(z)$  in a transducer by a piecewise constant function throughout the thickness.

Following this procedure, several layers of the same piezoelectric material, in the shape of disks 2.5 cm in diameter and of different thicknesses, have been bonded together with an epoxy glue. Thus a four-layer PZT transducer of overall

thickness  $e = 4$  mm, and a two-layer PVDF transducer of overall thickness  $e = 0.5$  mm, have been fabricated. For the PZT transducer, the electrodes were realized by silver painting the two external faces; for the PVDF transducer, the electrodes were constituted by two metallic films  $40 \mu\text{m}$  thick glued on both external faces.

The nonuniform distributions of the piezoelectric coefficient  $g_{33}(z)$ , which were implemented for these two transducers, are represented in Fig. 2, normalized to the maximum value  $g_{33}^{\text{max}}$  of the piezoelectric coefficient within the thickness.

For the PZT transducer, each layer has been poled separately in an external electric field, whose value was changed to achieve different degrees of poling. The value of  $g_{33}$  was measured for each of the four layers by recording the electric response of the piezoelectric layer under low-frequency mechanical excitation. From these measurements, a value of  $g_{33}^{\text{max}}$  of  $28 \times 10^{-3} \text{ V m}^{-1} \text{ Pa}^{-1}$  can be deduced for the PZT transducer.

The two layers of the PVDF transducer were made out of the same PVDF film, from two pieces mechanically ground to the desired thicknesses. We have thus assumed that the two layers have the same value of the piezoelectric coefficient  $g_{33} = g_{33}^{\text{max}}$ , with the change of sign in the distribution of Fig. 2 obtained simply by reversing the poling direction of one of them. For this type of PVDF films, the manufacturer's data indicate a value for  $g_{33}^{\text{max}}$  of  $480 \times 10^{-3} \text{ V m}^{-1} \text{ Pa}^{-1}$ .

For purposes of comparison, a PZT transducer and a PVDF transducer, both having a uniform distribution of the piezoelectric coefficient, were also realized. Each of these two transducers, made with a single layer of piezoelectric material, presents the same overall thickness and the same value of  $g_{33}^{\text{max}}$  as its nonuniform correspondent.

### B. Experimental results

The acoustoelectric properties of the transducers were studied when operating in water, with a low acoustic impedance backing, giving a reflection coefficient  $r_{23} = -1$  on the rear face.

The receive transfer function  $T(\omega)$  of a transducer was

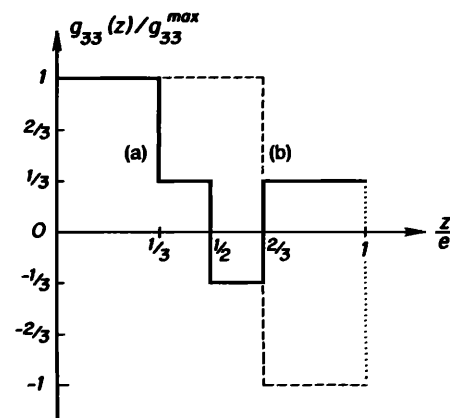


FIG. 2. Nonuniform distributions of the piezoelectric coefficient implemented in (a) the PZT transducer and (b) the PVDF transducer.

evaluated experimentally by placing it in a known acoustic field generated by a calibrated transmitting transducer in its farfield. The calibration of the transmitting transducer was achieved by using, as primary standard, a broadband calibrated hydrophone, the knowledge of the electric output of this calibrated hydrophone allowing to deduce, in absolute units of pressure, the acoustic field being probed.

To record the transfer functions, a wideband acoustic pulse was propagated to the transducer under test, while measuring the open circuit voltage resulting on its output. The electric signals on the input of the calibrated transmitter and on the output of the transducer under test, were digitized by a high-speed signal analyzer. The resulting numeric signals were transferred to a personal computer, which served to perform a spectral analysis of the signals and to apply the calibration curve of the transmitter. The transfer function of the receiver was thus deduced, in the frequency domain, as the ratio of the spectrum of its output voltage to the spectrum of the acoustic pulse incident on its front face.

For each of the four transducers tested, a theoretical form of the receive transfer function was also evaluated through a numerical evaluation of expression (1).

The results of these evaluations are presented in Figs. 3 and 4 for the two PZT transducers, and, in Figs. 5 and 6, for the two PVDF transducers. For each transducer, the experimental and the theoretical evaluations of the modulus  $|T(\omega)|$  of the transfer function have been plotted on the same graph versus frequency  $f = \omega / (2\pi)$ . The spectral resolution with which the experimental curves were recorded is 48 kHz for the PZT transducers and 96 kHz for the PVDF transducers. Each curve is depicted normalized to its maximum value  $|T_{\max}|$ .

For the uniform PZT transducer of Fig. 3(ex), measured  $|T_{\max}| = 0.44$  V/kPa; and for the nonuniform PZT transducer of Fig. 4(ex),  $|T_{\max}| = 0.21$  V/kPa. For the uniform PVDF transducer of Fig. 5(ex), it was found experimentally  $|T_{\max}| = 47$  mV/kPa; and for the nonuniform PVDF transducer of Fig. 6(ex),  $|T_{\max}| = 41$  mV/kPa.

To compute numerically from expression (1) the theoretical form of the different transfer functions, numerical

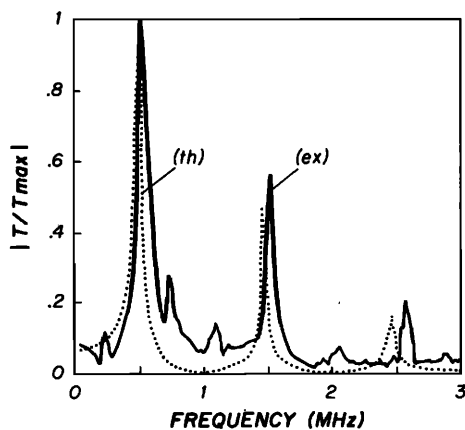


FIG. 3. Modulus of the receive transfer function of the uniform PZT transducer: (ex) experimental and (th) theoretical.

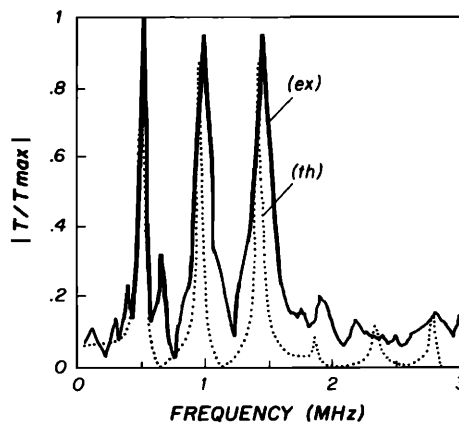


FIG. 4. Modulus of the receive transfer function of the nonuniform PZT transducer: (ex) experimental and (th) theoretical.

values are needed for the parameters  $c$ ,  $Z_2$ , and  $\alpha$  of the PZT and PVDF used in these experiments.

For PZT, the value of the velocity  $c$  has been deduced, through the formula  $c = 2ef_0$ , from the center frequency  $f_0 = 0.5$  MHz of the uniform transducer of Fig. 3(ex), and was found to be  $c = 4000$  m/s. With a density for PZT of  $7.8 \times 10^3$  kg/m<sup>3</sup>, follows the specific acoustic impedance:  $Z_2 = 31.2 \times 10^6$  kg m<sup>-2</sup> s<sup>-1</sup>. For the computations, we have taken  $\alpha = 0$ , neglecting the effect of attenuation in the PZT transducers.

The PVDF transducers tested in these experiments were fabricated with a PVDF of a porous type, which turned out to be the only kind of PVDF that could be successfully bonded by commonly available adhesives. From a Young's modulus of  $0.9 \times 10^9$  Pa given in the manufacturer's data, and a density of  $1.65 \times 10^3$  kg/m<sup>3</sup>, a velocity of sound  $c = 740$  m/s and a specific acoustic impedance  $Z_2 = 1.22 \times 10^6$  kg m<sup>-2</sup> s<sup>-1</sup> follow. The relatively low values for these two parameters come from the presence of air in the material, as a consequence of its porous structure. For the attenuation coefficient  $\alpha$  of this PVDF, we have taken a value of 12 dB/cm at 1 MHz with a quadratic frequency dependence. This attenuation coefficient is almost the attenuation coefficient of air.

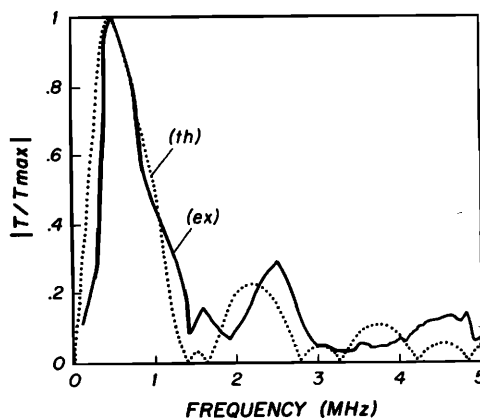


FIG. 5. Modulus of the receive transfer function of the uniform PVDF transducer: (ex) experimental and (th) theoretical.

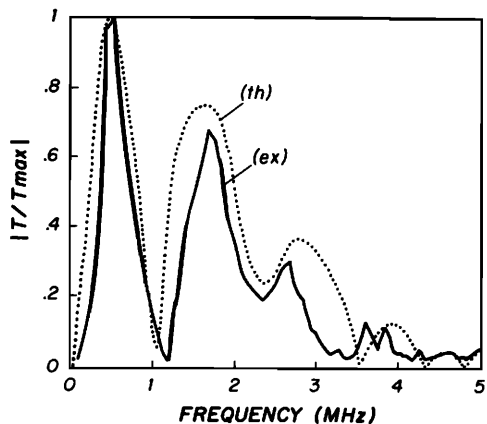


FIG. 6. Modulus of the receive transfer function of the nonuniform PDVF transducer: (ex) experimental and (th) theoretical.

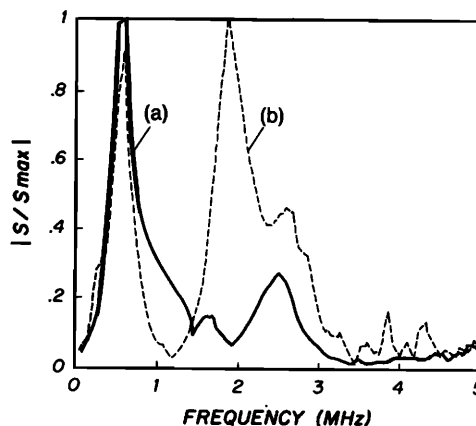


FIG. 8. Modulus of the transmit transfer functions of the (a) uniform and (b) nonuniform PVDF transducers.

It is well known, from bioacoustics for instance, that porous media, like bone for example, possess an attenuation coefficient very similar to that of air. However, for the relatively thin PVDF transducers employed here, the computational results show that the attenuation does not have a very important effect on the transfer function.

For further characterization, the transmit transfer function of each of the four transducers was also measured, using the calibrated hydrophone as a receiver in the farfield of the transmitter to be tested. After digitization and spectral analysis of the signals, the transmit transfer function  $S(\omega)$ —was evaluated, in the frequency domain, as the ratio of the spectrum of the acoustic pulse detected on the surface of the receiving hydrophone to the spectrum of the voltage on the input of the transmitter.

The results of these measurements are presented in Fig. 7 for the two PZT transducers, and in Fig. 8 for the two PVDF transducers. For each material, the modulus  $|S(\omega)|$  of the transmit transfer function of the uniform and of the nonuniform transducers are plotted on the same graph, versus frequency  $f = \omega/(2\pi)$ , with the same spectral resolutions as for the receiving mode, each curve being normalized to its maximum value  $|S_{\max}|$ .

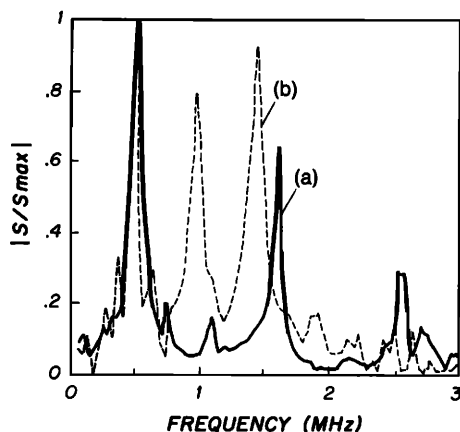


FIG. 7. Modulus of the transmit transfer functions of the (a) uniform and (b) nonuniform, PZT transducers.

For the uniform PZT transducer of Fig. 7(a), we measured  $|S_{\max}| = 3.4$  kPa/V; and for the nonuniform PZT transducer of Fig. 7(b),  $|S_{\max}| = 2.3$  kPa/V. For the uniform PVDF transducer of Fig. 8(a), it was found experimentally  $|S_{\max}| = 0.15$  kPa/V; and for the nonuniform PVDF transducer of Fig. 8(b),  $|S_{\max}| = 0.14$  kPa/V.

### C. Discussion of the results

First of all, we have to mention that, in the experiments reported here, which constitute an early attempt to characterize the behavior of nonuniform piezoelectric transducers, we did not seek to achieve very accurate measurements of the transfer functions. Instead, our goal was to examine, over a wide frequency range, if the main features predicted by the theoretical model were indeed present in the experimental results.

Two main factors limited the accuracy of the measurements. The first factor is that we had to utilize, in the test procedure, measuring transducers that were standard thickness mode transducers. These types of devices allow an important signal only in a relatively narrow frequency range. As we chose to examine a wide frequency range, extending typically up to ten times the center frequency of the tested transducers, we had to accept intervals of frequency with a low level of signal. The second limiting factor is that, to end up with measurements in absolute units, we had to make use, as absolute standard, of a calibrated hydrophone whose broad bandwidth trades off with low sensitivity.

However, high experimental accuracy was not necessary for our present purpose of getting a first experimental test of the theoretical description of nonuniform piezoelectric transducers. As we shall discuss below, the nonuniform transducers that were actually fabricated depart slightly in different points from the theoretical assumptions of the model, which renders them improper to furnish a highly accurate test of the theory.

The results of this section give an overview of the capability of the theoretical model to describe the behavior of transducers. The study of Sec. III will provide a better idea of the degree of accuracy of the description given by the theoretical model.

In the limits of experimental accuracy, the results presented in Figs. 3–6 show satisfactory agreement between the experimental and theoretical transfer functions. The main characteristics announced by the theoretical model concerning the shape of the transfer functions are observed in the experimental results.

For the uniform PZT transducer, of high acoustic impedance, the mechanical resonance modes are well separated, and a fundamental resonance peak and its third and fifth harmonics are recorded experimentally in the transfer function, and correctly predicted by the theoretical model.

For the uniform PVDF transducer, of low acoustic impedance, the mechanical resonance modes are less separated and overlap and interfere. A main resonance peak, followed by a second most important peak in the region of the fifth harmonic, are the main features observed experimentally in the transfer function, and are also present in the theoretical curve.

As we discussed in Ref. 1, the effect of a nonuniform distribution of the piezoelectric coefficient can be described as to retrieve back in the transfer function, resonance peaks corresponding to mechanical modes of vibration of the transducer, these peaks being weighted by the amplitude of the Fourier transform of the distribution of the piezoelectric coefficient at these frequencies.

This effect is clearly observed in the experimental results. For the PZT transducer of Fig. 4 (ex), with the nonuniform distribution of piezoelectric coefficient of Fig. 2(a), as predicted by the theory, the outcome is a transfer function in which the resonant peak corresponding to the second harmonic mode of vibration has been restored, and a redistribution of amplitude has led to three first resonance peaks of comparable magnitude.

Similarly, for the PVDF transducer of Fig. 6, the effect of the nonuniform distribution of Fig. 2(b) is to promote the resonance peak present in the region of the third harmonic.

For comparing the absolute magnitude of the experimental and theoretical transfer functions, one has to bear in mind that, for a given profile of the distribution of the piezoelectric coefficient, the overall amplitude of the theoretical transfer function is directly proportional to the value of  $g_{33}^{\max}$ . With the values of  $g_{33}^{\max}$  given in Sec. II A, for PZT, the theoretical calculation of the transfer function gives, for the uniform transducer,  $|T_{\max}| = 1.5 \text{ V/kPa}$ , and for the nonuniform transducer,  $|T_{\max}| = 1.0 \text{ V/kPa}$ . For PVDF, it follows theoretically, for the uniform transducer,  $|T_{\max}| = 0.31 \text{ V/kPa}$ , and, for the nonuniform transducer,  $|T_{\max}| = 0.23 \text{ V/kPa}$ .

It can be observed that the ratio of the value of  $|T_{\max}|$  for the uniform transducer to the value of  $|T_{\max}|$  for the corresponding nonuniform transducer, is found experimentally, for both materials, to be roughly of the same order as the ratio predicted theoretically. However, the absolute value of  $|T_{\max}|$  determined experimentally is always found smaller than the absolute value of  $|T_{\max}|$  predicted theoretically. The discrepancy amounts approximately to a factor of 4 for the PZT transducers, and to a factor of 6 for the PVDF transducers.

An explanation for this observation could be a possible

variation of the piezoelectric coefficient with frequency. The usual method for measuring the piezoelectric coefficient of a material is to record its voltage response when subjected to a mechanical excitation of low frequency, commonly in the region of 100 Hz or below. This technique leads for the piezoelectric coefficient to values reported in Sec. II A, and that were utilized for the theoretical calculation of the transfer functions. However, these values of the piezoelectric coefficient determined in the low-frequency range, may not be the appropriate values to describe transducers operating in the frequency region of 1 MHz. Piezoelectric activity, which involves induction and relaxation of electric dipoles, can reasonably be expected to decrease as frequency increases, leading to smaller values for the piezoelectric coefficient in the high-frequency range.<sup>2</sup> We note that, in the theoretical expression of the transfer function as given by (1), an explicit frequency dependence of the piezoelectric coefficient  $g_{33}$ , is readily taken into consideration.

Finally, to account for the remaining slight differences that are still observed between the experimental and theoretical transfer functions, various factors can be evoked. As we already mentioned, the precision, and also the finite spectral resolution, with which the measurements were performed constituted a natural limitation in the recording of the transfer functions. Some departures between the actual constitution of the experimental transducers and the hypothesis of their description can also play a role. For instance, in practice, the lateral dimensions of the transducers are not infinite, while the theoretical model assumes a laterally unbounded transducer. Also, the influence of the electrodes was neglected, although it could have been included, in the theoretical derivation of expression (1). For the nonuniform transducers, the layers of adhesive bonding the piezoelectric layers together, although very thin, can cause a perturbation in the distributions of the piezoelectric coefficient and acoustic pressure in the transducers. Finally, we have assumed that, to describe the experimental transducers, in each single layer of the multilayered structure the piezoelectric coefficient was uniform throughout the thickness. The poling process that builds up piezoelectric activity in a layer of material, which consists of submitting the layer to an external electric field considered uniform, makes this assumption of a uniform piezoelectric coefficient reasonable. But this assumption can also reasonably be viewed as approximate, and small departure from a uniform distribution of the piezoelectric coefficient in the thickness of a layer can also be envisaged.

Now, for the transmitting mode of the transducers, the results of Figs. 7 and 8 show that the nonuniform distributions of the piezoelectric coefficient produce, in the transmit transfer functions, changes similar to those observed in the receive transfer functions. Peaks in the frequency regions corresponding to different harmonics of the mechanical vibration spectrum of the transducers are set up in the transfer functions.

The theoretical model we developed, as it is now, does not describe the transmit transfer function of a nonuniform transducer, not does any other model known at present. However, as we mentioned in Ref. 1, and as it is demonstrat-

ed experimentally here, the reciprocity theorem allows one to assume that any feature predicted by the model for the receiving mode of a nonuniform transducer will have its analog in the transmitting mode.

### III. NUMERICAL EVALUATION OF BACKING MATERIALS

To gain further insight into the ability of general equation (1) to describe accurately the receive transfer function of a piezoelectric transducer, we have studied the effects on the transfer function that are predicted by (1) when different backing media are employed with the transducer.

For a uniform ceramic transducer, associated successively with six different backing materials characterized by their specific acoustic impedance  $Z_3$ , we have evaluated numerically from (1) the theoretical form of the transfer function. These results have been compared to the transfer functions reported by Kossoff<sup>3</sup> and describing the same transducers.

The results of Kossoff, which appear in Fig. 6 of Ref. 3, were also evaluated numerically, but through the use of the now classical Mason equivalent circuit of a piezoelectric transducer.<sup>4</sup> The two approaches considered, exhibit quite different mathematical formulations.

We have applied in Eq. (1) the exact numerical values defined by Kossoff for the parameters of the transducer. The studied transducer, with a thickness  $e = 1.05$  mm, was made of a PZT7A ceramic having a velocity of sound  $c = 4830$  m/s, a specific acoustic impedance  $Z_2 = 35.7 \times 10^6$  kg m<sup>-2</sup> s<sup>-1</sup>, and in which the effect of attenuation was neglected. The specific acoustic impedance  $Z_1$  of the propagating medium was that of water. Six different values of the specific acoustic impedance  $Z_3$  of the backing material have then been investigated, ranging from  $Z_3 = 4 \times 10^2$  kg m<sup>-2</sup> s<sup>-1</sup> for air to  $Z_3 = 45 \times 10^6$  kg m<sup>-2</sup> s<sup>-1</sup> for nickel.

By applying these conditions in Eq. (1), a maximum value for the different transfer functions was found at a frequency  $f_0 = 2.3$  MHz, as was also reported by Kossoff. The modulus of the transfer functions, computed with Eq. (1) for the six different cases, are plotted in Fig. 9, with the value

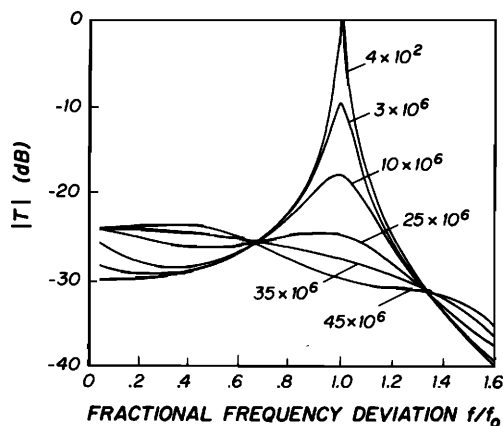


FIG. 9. Modulus of the receive transfer functions, calculated with the theoretical model, for a uniform ceramic transducer associated with different backing materials, whose acoustic impedances are given in kg m<sup>-2</sup> s<sup>-1</sup>.

of  $Z_3$  in each case indicated on the corresponding curve. The - dB reference level has been chosen for all the curves as the maximum value of the transfer function of the air-backed transducer.

For comparison, we have reproduced in Fig. 10 the results obtained by Kossoff.

The results of Fig. 9, when compared to the results from Kossoff, show an almost exact correspondence between the two sets of data, both in the shape and magnitude of the transfer functions, in all the cases considered. This outcome indicates that, in the situations tested here, the theoretical transfer functions given by Eq. (1) are equivalent to the transfer functions derived from Mason's equivalent circuit. From this equivalence between the two theoretical descriptions of the transducer, many experimental verifications that have now been made of Mason's equivalent circuit can logically be considered as also providing evidence for the validity of the new model we propose.

### IV. CONCLUSION

In this paper, we have reviewed results of a theoretical analysis published previously,<sup>1</sup> which provides a model to describe theoretically the receive transfer function of an acoustoelectric transducer having a nonuniform distribution of the piezoelectric coefficient throughout its bulk. An experimental test of the theoretical model has been conducted. Nonuniform piezoelectric transducers were fabricated, their transfer functions were measured, and it was shown that the main features observed experimentally in the transfer functions are correctly predicted by the theoretical model. A numerical study of the influence of the backing medium of a transducer was also undertaken, which revealed an equivalence between the predictions of the theoretical model and the predictions which follow from the Mason equivalent circuit of the transducer.

So, at this point, several pieces of evidence are gathered in favor of the validity of the theoretical model we propose: First, this model comes as a direct consequence of solid first principles; second, in the situations tested, its predictions are found equivalent to those of Mason's equivalent circuit; and, third, in the limit of experimental accuracy, it has been shown to describe satisfactorily the transfer functions recorded experimentally. For the receiving mode of a piezo-

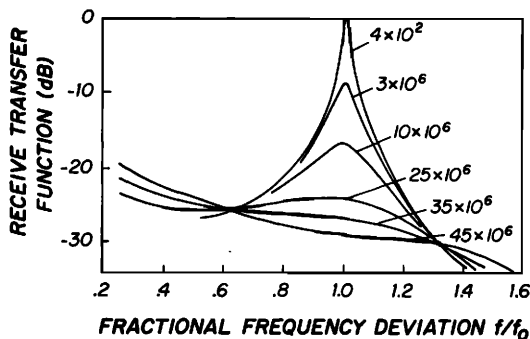


FIG. 10. Modulus of the receive transfer functions following from the Mason equivalent circuit. (Redrawn after data from Kossoff.<sup>3</sup>)

electric transducer, this model provides a quite general theoretical description. It describes the electric signals generated by the transducer terminated in any arbitrary electric load, it includes the attenuation in the piezoelectric material as well as the influence of the backing medium, and, in the theoretical framework developed in Ref. 1, effects of multiple matching layers are readily taken into account. In some aspect, the model is more general than Mason's equivalent circuit, for it describes explicitly the influence of a nonuniform distribution of the piezoelectric coefficient within the transducer.

As we discussed in Ref. 1, the theoretical analysis shows that the nonuniform distribution of the piezoelectric coefficient provides a means to optimize the transfer function of the transducer. The results of the present paper demonstrate that the theoretical predictions of the model are indeed observed experimentally. So, it now becomes possible to design nonuniform piezoelectric transducers, with different optimal shapes of transfer function, depending on the required applications. The physical realization of these nonuniform transducers, guided by the theoretical model, will naturally depend on the possibilities and limitations of actual implementation of nonuniform distributions of the piezoelectric

coefficient in materials, and to draw full benefit from this new type of transducer may require developments in processing techniques of piezoelectric materials.

## ACKNOWLEDGMENTS

The authors thank Steven Orwoll for the illustrations and Elaine Quarve for secretarial assistance. Funding for this work was provided by Grant No. CA43920 from the National Institutes of Health.

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