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Intrication quantique, corrélations quantiques, et traitement de l'information.

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1 – Quantum basics

- Quantum state : A vector $|\psi\rangle$ with unit norm in a complex Hilbert space $\mathcal{H}$.
- Quantum measurement : As a random projection of $|\psi\rangle$ in an orthonormal basis of $\mathcal{H}$.
- Quantum evolution : By a unitary operator U acting as $|\psi\rangle \longmapsto \mathrm{U}|\psi\rangle$.

Quantum system in dimension 2 : the qubit

(photon, electron, ...)

State vector $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$
in some orthonormal basis $\{|0\rangle, |1\rangle\}$ of $\mathcal{H}_2$,
with complex $\alpha, \beta \in \mathbb{C}$ such that $|\alpha|^2 + |\beta|^2 = \langle\psi|\psi\rangle = \|\psi\|^2 = 1$.

$|\psi\rangle = \begin{bmatrix}\alpha\\ \beta\end{bmatrix}$, $|\psi\rangle^\dagger = \langle\psi| = [\alpha^*, \beta^*]$ $\Longrightarrow \langle\psi|\psi\rangle = \|\psi\|^2 = |\alpha|^2 + |\beta|^2$ scalar.

Measurement of the qubit

When a qubit in state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$
is measured in the orthonormal basis $\{|0\rangle, |1\rangle\}$,

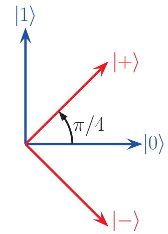
$\Longrightarrow$ only 2 possible outcomes (Born rule) :
state $|0\rangle$ with probability $|\alpha|^2 = |\langle 0|\psi\rangle|^2 = \langle 0|\psi\rangle\langle\psi|0\rangle = \langle\psi|0\rangle\langle 0|\psi\rangle$, or
state $|1\rangle$ with probability $|\beta|^2 = |\langle 1|\psi\rangle|^2 = \langle 1|\psi\rangle\langle\psi|1\rangle = \langle\psi|1\rangle\langle 1|\psi\rangle$.

Quantum measurement : usually :
- a probabilistic process,
- as a destructive projection of the state $|\psi\rangle$ in an orthonormal basis,
- with statistics evaluable over repeated experiments with same preparation $|\psi\rangle$.

Hadamard basis

Another orthonormal basis of $\mathcal{H}_2$

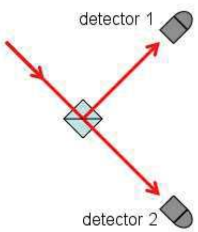
$$\left\{ |+\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big) ; \quad |-\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle - |1\rangle\big) \right\}.$$



$\Longleftrightarrow$ Computational orthonormal basis

$$\left\{ |0\rangle = \frac{1}{\sqrt{2}}\big(|+\rangle + |-\rangle\big) ; \quad |1\rangle = \frac{1}{\sqrt{2}}\big(|+\rangle - |-\rangle\big) \right\}.$$

Experimental implementation



Two states of polarization of a photon :
polarizing beam splitter

Rotating the polarizer changes the measurement basis.

Multiple qubits

A system of L qubits has its state in the tensor-product space $\mathcal{H}_2^{\otimes L}$,
with dimension 2^L.

Qubit pair $L = 2$ **:** in space $\mathcal{H}_2 \otimes \mathcal{H}_2 \equiv \mathcal{H}_2^{\otimes 2}$, with dimension $2^2 = 4$,
and orthonormal basis : $|0\rangle \otimes |0\rangle \equiv |0\rangle|0\rangle \equiv |00\rangle$,
$|0\rangle \otimes |1\rangle \equiv |0\rangle|1\rangle \equiv |01\rangle$,
$|1\rangle \otimes |0\rangle \equiv |1\rangle|0\rangle \equiv |10\rangle$,
$|1\rangle \otimes |1\rangle \equiv |1\rangle|1\rangle \equiv |11\rangle$.

General state of a qubit pair : $|\psi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$.

A special separable state
$$|\phi\rangle = \big(\alpha_1|0\rangle + \beta_1|1\rangle\big) \otimes \big(\alpha_2|0\rangle + \beta_2|1\rangle\big) = \alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\beta_2|11\rangle.$$

A multipartite state which is not separable is entangled.

An entangled state behaves as a nonlocal whole : what is done on one part may influence the other part instantly, no matter how distant they are.

- Example of a separable state of two qubits AB :

$$|\psi_{AB}\rangle = |+\rangle \otimes |+\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big) \otimes \frac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big) = \frac{1}{2}\big(|00\rangle + |01\rangle + |10\rangle + |11\rangle\big).$$

When measured in the basis $\{|0\rangle, |1\rangle\}$, each qubit A and B can be found in state $|0\rangle$ or $|1\rangle$ independently with probability $1/2$.

- Example of an entangled state of two qubits AB :

$$|\psi_{AB}\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle\big) = \frac{1}{\sqrt{2}}\big(|00\rangle + |11\rangle\big).$$

When measured in the basis $\{|0\rangle, |1\rangle\}$, each qubit A and B can be found in state $|0\rangle$ or $|1\rangle$ with probability $1/2$ (randomly, no predetermination before measurement).
But when A is found in $|0\rangle$ necessarily B is found in $|0\rangle$,
and when A is found in $|1\rangle$ necessarily B is found in $|1\rangle$,
no matter how distant the two qubits are before measurement.

But alternatively, measurement can be done in basis $\{|+\rangle, |-\rangle\}$ ...

$$|\psi_{AB}\rangle = \frac{1}{\sqrt{2}}\big(|00\rangle + |11\rangle\big) = \frac{1}{\sqrt{2}}\big(|{+}{+}\rangle + |{-}{-}\rangle\big).$$

When measured in the basis $\{|+\rangle, |-\rangle\}$, each qubit A and B can be found in state $|+\rangle$ or $|-\rangle$ with probability $1/2$ (randomly, no predetermination before measurement).
But when A is found in $|+\rangle$ necessarily B is found in $|+\rangle$,
and when A is found in $|+\rangle$ necessarily B is found in $|-\rangle$,
no matter how distant the two qubits are before measurement.

Just as if B was instantly informed that measurement of A had taken place in $\{|+\rangle, |-\rangle\}$ and not in $\{|0\rangle, |1\rangle\}$, and it should adjust its state as $|\pm\rangle$ according to the measurement result for A.

$\Longrightarrow$ EPR paradox (Einstein-Podolski-Rosen) :

[1] A. Einstein, B. Podolsky, N. Rosen ; "Can quantum-mechanical description of physical reality be considered complete ?"; *Physical Review* 47, 777–780 (1935).

2 – Entanglement & quantum correlations

- Einstein-Podolsky-Rosen : Quantum mechanics might be incomplete (1935).
- If hidden variables exist $\Longrightarrow$ Bell inequalities are satisfied (1964).
- A. Aspect experiments : Bell inequalities are violated by Reality (1982).

Quantum correlations (1/5)

For any four random binary variables A_1, A_2, B_1, B_2 with values ± 1,
$\Gamma = (A_1 - A_2)B_1 - (A_1 + A_2)B_2 = A_1B_1 - A_2B_1 - A_1B_2 - A_2B_2 = \pm 2$,
because since $A_1, A_2 = \pm 1$, either $(A_1 - A_2)B_1 = 0$ or $(A_1 + A_2)B_2 = 0$,
and in each case the remaining term is ± 2.

So for any probability distribution on (A_1, A_2, B_1, B_2), the average
$\langle\Gamma\rangle = \big\langle A_1B_1 - A_2B_1 - A_1B_2 - A_2B_2\big\rangle = \langle A_1B_1\rangle - \langle A_2B_1\rangle - \langle A_1B_2\rangle - \langle A_2B_2\rangle$
necessarily verifies $-2 \le \langle\Gamma\rangle \le 2$. Bell inequalities (1964).

The binary variables at ± 1 will be obtained (by Alice and Bob)
from the results when measuring a qubit
of a qubit pair AB prepared in a joint state $|\psi_{AB}\rangle$.

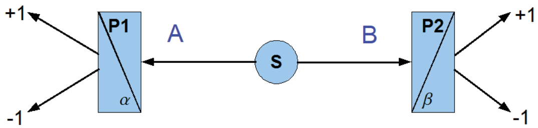
[2] J. S. Bell ; "On the Einstein–Podolsky–Rosen paradox"; *Physics* 1, 195–200 (1964).
[3] **J. F. Clauser**, M. A. Horne, A. Shimony, R. A. Holt ; "Proposed experiment to test local hidden-variable theories"; *Physical Review Letters* 23, 880–884 (1969).

Quantum correlations (2/5)

Alice or Bob obtains results ± 1 by measuring her / his qubit in basis
$\big\{|\lambda_+(\theta)\rangle = [\cos(\theta/2), \sin(\theta/2)]^\top \ , \ |\lambda_-(\theta)\rangle = [-\sin(\theta/2), \cos(\theta/2)]^\top\big\}$.

Alice measures at $\theta = \alpha$ to obtain $A = \pm 1$, and Bob measures at $\theta = \beta$ to obtain $B = \pm 1$,
with the joint probabilities $P(A = \pm 1, B = \pm 1) = \big|\big\langle \lambda_\pm(\alpha) \otimes \lambda_\pm(\beta) \,|\, \psi_{AB}\big\rangle\big|^2$.

Alice and Bob share a qubit pair AB in the entangled state $|\psi_{AB}\rangle = \frac{1}{\sqrt{2}}\big(|01\rangle - |10\rangle\big)$.



Quantum correlations (3/5)

$\Longrightarrow$ Joint probabilities

$$P(A = +1, B = +1) = P(A = -1, B = -1) = \frac{1}{4}\big[1 - \cos(\alpha - \beta)\big] ,$$

$$P(A = +1, B = -1) = P(A = -1, B = +1) = \frac{1}{4}\big[1 + \cos(\alpha - \beta)\big] ,$$

and by summation the marginal probabilities

$$P(A = +1) = P(A = -1) = P(B = +1) = P(B = -1) = \frac{1}{2} ,$$

and the correlation $\langle AB\rangle = -\cos(\alpha - \beta)$.

Quantum correlations (4/5)

To obtain four binary variables ± 1 ,
Alice randomly switches between measuring A_1 when $\theta = \alpha_1$ or A_2 when $\theta = \alpha_2$,
Bob randomly switches between measuring B_1 when $\theta = \beta_1$ or B_2 when $\theta = \beta_2$.

For $\langle\Gamma\rangle = \langle A_1B_1\rangle - \langle A_2B_1\rangle - \langle A_1B_2\rangle - \langle A_2B_2\rangle$ one obtains
$\langle\Gamma\rangle = -\cos(\alpha_1 - \beta_1) + \cos(\alpha_2 - \beta_1) + \cos(\alpha_1 - \beta_2) + \cos(\alpha_2 - \beta_2)$.

The choice $\alpha_1 = 0$, $\alpha_2 = \pi/2$ and $\beta_1 = 3\pi/4$, $\beta_2 = \pi/4$ leads to
$\langle\Gamma\rangle = -\cos(3\pi/4) + \cos(\pi/4) + \cos(\pi/4) + \cos(\pi/4) = 2\sqrt{2} > 2$.

Bell inequalities are violated by quantum correlations !!

Experimentally verified (Aspect *et al.*, Phys. Rev. Lett. 1981, 1982.) Nobel 2022

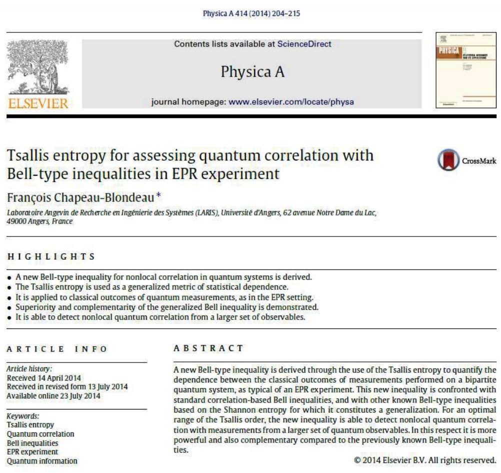
[4] **A. Aspect**, P. Grangier, G. Roger ; "Experimental test of realistic theories via Bell's theorem"; *Physical Review Letters* 47, 460–463 (1981).

Quantum correlations (5/5)

No possibility of hidden-variables theories underneath quantum mechanics.

Confirmation of quantum mechanics replacing local realism and separability (classical) by a nonlocal non-separable reality (quantum).

Quantities that are not actually measured have no physical reality.



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Tsallis entropy for assessing quantum correlation with Bell-type inequalities in EPR experiment

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HIGHLIGHTS

- A new Bell-type inequality for nonlocal correlation in quantum systems is derived.
- The Tsallis entropy is used as a generalized metric of statistical dependence.
- It is applied to classical outcomes of quantum measurements, as in the EPR setting.
- Superiority and complementarity of the generalized Bell inequality is demonstrated.
- It is able to detect nonlocal quantum correlation from a larger set of observables.

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ABSTRACT

A new Bell-type inequality is derived through the use of the Tsallis entropy to quantify the dependence between the classical outcomes of measurements performed on a bipartite quantum system, as typical of an EPR experiment. This new inequality is confronted with standard correlation-based Bell inequalities, and with other known Bell-type inequalities based on the Shannon entropy for which it constitutes a generalization. For an optimal range of the Tsallis order, the new inequality is able to detect nonlocal quantum correlation with measurements from a larger set of quantum observables. In this respect it is more powerful and also complementary compared to the previously known Bell-type inequalities.

3 – Exploiting entanglement

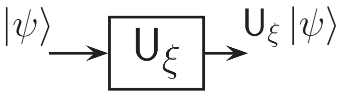
- Teleportation.

[5] D. Bouwmeester, J.-W. Pan, K. Mattle, M. Eibl, H. Weinfurter, **A. Zeilinger** ; "Experimental quantum teleportation"; *Nature* 390, 575–579 (1997).

- Parameter estimation on a quantum system.
- Communication over a noisy quantum channel.

• Quantum parameter estimation

An excitation signal $|\psi\rangle$ is prepared at the input,
to probe the quantum process U_ξ depending on a parameter ξ.
From output signal $\mathsf{U}_\xi|\psi\rangle$ estimate ξ.

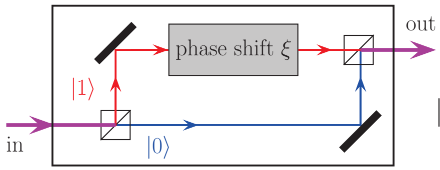


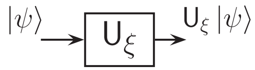
$\Longrightarrow$ Quantum sensing and metrology
for high sensitivity, high accuracy.

[6] C. L. Degen, *et al.*; "Quantum sensing"; *Reviews of Modern Physics* 89, 035002,1–39 (2017).
[7] V. Giovannetti, *et al.*; "Advances in quantum metrology"; *Nature Photonics* 5, 222–229 (2011).

Un photon (qubit) selon sa polarisation est transformé par un interféromètre :

$|\psi\rangle \longmapsto \mathsf{U}_\xi|\psi\rangle$, avec l'opérateur unitaire $\mathsf{U}_\xi = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\xi} \end{bmatrix} = |0\rangle\langle 0| + e^{i\xi}|1\rangle\langle 1|$.





Le problème : Estimer (efficacement (optimalement)) la valeur du déphasage ξ.
Quel signal d'excitation $|\psi\rangle$ ou $\rho = |\psi\rangle\langle\psi|$ en entrée ?
Quels mesure quantique et traitement du signal de sortie $\mathsf{U}_\xi|\psi\rangle$ ou $\rho_\xi = \mathsf{U}_\xi|\psi\rangle\langle\psi|\mathsf{U}_\xi^\dagger$?
Comment évaluer l'efficacité ?

On se guide sur l'information de Fisher. [8]

• En classique, à partir de mesures $\vec{x}$, tout estimateur $\widehat{\xi}$ pour ξ
possède une erreur quadratique moyenne $\langle(\widehat{\xi} - \xi)^2\rangle$
minorée via l'information de Fisher classique $F_c(\xi) = \left\langle \left[\partial_\xi \ln P(\vec{x};\xi)\right]^2 \right\rangle$,
assurant $\langle(\widehat{\xi} - \xi)^2\rangle \geq$ borne de Cramér-Rao $\sim \dfrac{1}{F_c(\xi)}$,
avec l'estimateur du maximum de vraisemblance qui sature la borne, à $\vec{x}$ grand.

• En quantique, pour des données $\vec{x}$ issues de mesures quantiques sur un état ρ_ξ,
on a $F_c(\xi)$ majorée par l'information de Fisher quantique $F_q(\xi) = \left\langle \left[\mathcal{D}_\xi \rho_\xi\right]^2 \right\rangle$,
(avec $\mathcal{D}_\xi$ dérivée logarithmique symétrisée) assurant $F_c(\xi) \leq F_q(\xi)$,
et $F_q(\xi) = 2\sum_{j,k} \dfrac{|\langle\lambda_j|\partial_\xi\rho_\xi|\lambda_k\rangle|^2}{\lambda_j + \lambda_k}$, via décomposition spectrale $\left\{\lambda_j, |\lambda_j\rangle\right\}$ de ρ_ξ.

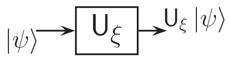
[8] O. E. Barndorff-Nielsen, R. D. Gill; "Fisher information in quantum statistics"; *Journal of Physics A* 33, 4481–4490 (2000).

Optimal strategy

• Optimal input $|\psi\rangle = \dfrac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big) = |+\rangle \Longrightarrow$ output $\mathsf{U}_\xi|\psi\rangle = \dfrac{1}{\sqrt{2}}\big(|0\rangle + e^{i\xi}|1\rangle\big)$

optimally measured in basis $\big\{|+\rangle, |-\rangle\big\}$

to yield $\Pr\big\{|+\rangle\big\} = \big|\langle +|\mathsf{U}_\xi|\psi\rangle\big|^2 = \dfrac{1+\cos(\xi)}{2}$.

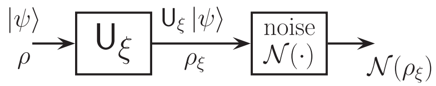


• N successive experiments deliver a sequence of
N_+ outcomes $|+\rangle$ and $N_- = N - N_+$ outcomes $|-\rangle$.

• From the measured data (N_+, N_-),
the value of ξ is estimated by an estimator $\widehat{\xi} = \widehat{\xi}(N_+, N_-)$.
Maximum likelihood estimator $\widehat{\xi}(N_+, N_-) = \arg\max_\xi \Pr(N_+, N_-; \xi)$

$\Longrightarrow \widehat{\xi} = \arccos\left(\dfrac{N_+ - N_-}{N}\right)$.

Prise en compte du bruit quantique ou décohérence



État pur $|\psi\rangle \longrightarrow$ état mélangé d'opérateur densité $\rho = \sum_j p_j |\psi_j\rangle\langle\psi_j|$.

Modélisation du bruit quantique comme une évolution non unitaire
$\rho \longmapsto \mathcal{N}(\rho) = \sum_\ell \mathsf{V}_\ell\,\rho\,\mathsf{V}_\ell^\dagger$
avec les opérateurs de Kraus V_ℓ vérifiant $\sum_\ell \mathsf{V}_\ell^\dagger \mathsf{V}_\ell = \mathrm{Id}$,
qui caractérisent le bruit quantique en présence.

[9] S. Haroche, J.-M. Raimond; "Exploring the Quantum : Atoms, Cavities and Photons"; *Oxford University Press*, 2006.

Bruit de bit-flip $\rho \longmapsto \mathcal{N}(\rho) = (1-p)\rho + p\sigma_x\rho\sigma_x^\dagger$
Bruit de phase-flip $\rho \longmapsto \mathcal{N}(\rho) = (1-p)\rho + p\sigma_z\rho\sigma_z^\dagger$
Bruit dépolarisant $\rho \longmapsto \mathcal{N}(\rho) = (1-p)\rho + \dfrac{p}{3}\big(\sigma_x\rho\sigma_x^\dagger + \sigma_y\rho\sigma_y^\dagger + \sigma_z\rho\sigma_z^\dagger\big)$

$\Longrightarrow$ le signal d'excitation $|+\rangle = \dfrac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big)$ reste optimal en entrée,
mais la mesure projective dans la base $\big\{|+\rangle, |-\rangle\big\}$ n'est plus optimale en sortie.

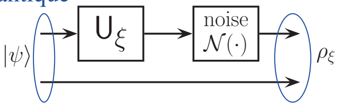
Bruit thermique quantique $\rho \longmapsto \mathcal{N}(\rho) = \sum_{\ell=1}^{4} \Lambda_\ell\,\rho\,\Lambda_\ell^\dagger$

$\Longrightarrow$ le signal d'entrée $|+\rangle = \dfrac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big)$ n'est plus toujours optimal.

[10] F. Chapeau-Blondeau; "Optimizing qubit phase estimation"; *Physical Review A* 94, 022334,1–14 (2016).
[11] F. Chapeau-Blondeau; "Optimized probing states for qubit phase estimation with general quantum noise"; *Physical Review A* 91, 052310,1–13 (2015).

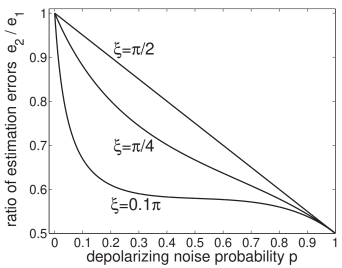
Amélioration par l'intrication quantique

Paire de qubits intriqués en entrée
d'état $|\psi\rangle = \dfrac{1}{\sqrt{2}}\big(|00\rangle + |11\rangle\big)$



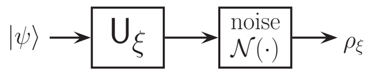
avec un seul qubit actif qui interagit avec le processus U_ξ à estimer.

La présence du qubit intriqué passif,
bien qu'il n'interagisse pas
avec le processus U_ξ à estimer,
toujours réduit l'erreur d'estimation,
en présence de bruit. ☺



Amélioration par l'intrication quantique

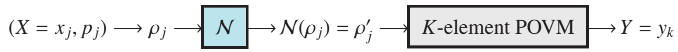
… ou avec deux ou plusieurs qubits intriqués actifs
pour interagir avec le processus U_ξ à estimer.



On obtient des bénéfices pour l'estimation quantique.
Mais les états intriqués d'entrée $|\psi\rangle$ optimaux, et leur traitement optimal,
selon les types de bruit, sont loin d'être tous caractérisés …

[12] F. Chapeau-Blondeau; "Optimized entanglement for quantum parameter estimation from noisy qubits"; *International Journal of Quantum Information* 16, 1850056,1–25 (2018).
[13] F. Chapeau-Blondeau; "Entanglement-assisted quantum parameter estimation from a noisy qubit pair: A Fisher information analysis"; *Physics Letters A* 381, 1369–1378 (2017).
[14] N. Gillard, E. Belin, F. Chapeau-Blondeau; "Estimation quantique en présence de bruit améliorée par l'intrication"; Actes du 26ème Colloque GRETSI sur le Traitement du Signal et des Images, Juan-les-Pins, France, 5–8 sept. 2017.

• Communication over a noisy quantum channel (1/3)



Input–output mutual information $I(X;Y) \leq \chi(\rho'_j, p_j) = S(\rho') - \sum_{j=1}^{J} p_j S(\rho'_j)$,

with quantum entropy $S(\rho) = -\mathrm{tr}\big(\rho\log(\rho)\big)$, and $\rho' = \sum_{j=1}^{J} p_j\rho'_j$.

Holevo information $\chi(\rho'_j, p_j)$ is an achievable rate for error-free communication,
by coding successive classical input symbols X in blocks of length $L \to \infty$.

[15] B. Schumacher, M. D. Westmoreland; "Sending classical information via noisy quantum channels"; *Physical Review A* 56, 131–138 (1997) .
[16] A. S. Holevo; "The capacity of the quantum channel with general signal states"; *IEEE Transactions on Information Theory* 44, 269–273 (1998).

Communication over a noisy quantum channel (2/3)

For given $\mathcal{N}(\cdot)$ therefore $\chi_{\max}(\mathcal{N}) = \max_{\{p_j, \rho_j\}} \chi\big(\mathcal{N}(\rho_j), p_j\big)$

is the overall maximum and achievable rate for error-free communication
of classical information over a noisy quantum channel,
or the classical information capacity of the quantum channel,
for product states or successive independent uses of the channel.

NB : The Holevo information capacity $\chi_{\max}(\mathcal{N})$ can be achieved by
no more than N^2 *pure* input states $\rho_j = |\psi_j\rangle\langle\psi_j|$ with $|\psi_j\rangle \in \mathcal{H}_N$.
[Shor, *J. Math. Phys.* 43 (2002) 4334. Shor, *Com. Math. Phys.* 246 (2004) 453.]

Communication over a noisy quantum channel (3/3)

For product states or successive independent uses of the channel
the Holevo capacity is additive $\chi_{\max}(\mathcal{N}_1 \otimes \mathcal{N}_2) = \chi_{\max}(\mathcal{N}_1) + \chi_{\max}(\mathcal{N}_2)$.

For non-product states or successive non-independent but entangled uses of the channel,
the Holevo information is superadditive $\chi_{\max}(\mathcal{N}_1 \otimes \mathcal{N}_2) \geq \chi_{\max}(\mathcal{N}_1) + \chi_{\max}(\mathcal{N}_2)$.

For many channels it is found additive, $\chi_{\max}(\mathcal{N}_1 \otimes \mathcal{N}_2) = \chi_{\max}(\mathcal{N}_1) + \chi_{\max}(\mathcal{N}_2)$
so that entanglement does not improve over the product-state capacity.

Yet for some channels it has been found strictly superadditive,
$\chi_{\max}(\mathcal{N}_1 \otimes \mathcal{N}_2) > \chi_{\max}(\mathcal{N}_1) + \chi_{\max}(\mathcal{N}_2)$ meaning that entanglement does improve
over the product-state capacity.

M. B. Hastings; "Superadditivity of communication capacity using entangled inputs";
Nature Physics 5 (2009) 255–257.

Then, which channels ? which entanglements ? which improvement ?
which capacity ? . . . (largely, these are open issues).

Outlook

- **Quantum entanglement** shows specific distinctive potential
for quantum information, quantum computation, quantum signal processing.

- Not completely understood theoretically, and exploited practically.

- Especially in the presence of quantum noise or decoherence.

Merci de votre attention.